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# Performance Analysis of V2V and V2I LiFi Communication Systems in Traffic Lights

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*Research Article*

# Performance Analysis of V2V and V2I LiFi Communication Systems in Traffic Lights

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Vehicular networks is a key technology for efficiently communicating both user's devices and cars for timely information regarding safe driving conditions and entertaining applications like social media, video streaming, and gaming services, among others. In view of this, mobile communications making use of cellular resources may not be an efficient and cost-effective alternative. In this context, the implementation of light-fidelity (LiFi) in vehicular communications could be a low-cost, high-data-rate, and efficient-bandwidth usage solution. In this work, we propose a mathematical analysis to study the average throughput in a road intersection equipped with a traffic light that operates as a server, which is assumed to have LiFi communication links with the front lights of the vehicles waiting for the green light. We further assume that the front vehicle (the car next to the traffic light) is able to communicate to the car immediately behind it by using its own tail lights and the front lights of such vehicle, and so on and so forth. The behavior of the road junction is modeled by a Markov chain, applying the Queueing theory with an M/M/1 system in order to obtain the average queue length. Then, Little's theorem is applied to calculate the average waiting delay when the red light is present in the traffic light. Finally, the mathematical expression of the data throughput is derived.

## 1. Introduction

Nowadays, a major open issue in most cities is to improve traffic conditions by means of continuous surveillance of drivers. Manual traffic monitoring is a classical approach that is costly and inefficient since a high number of human resources are needed and only some vehicular crossings can be covered at partial times. To tackle this issue, the authors in [1] proposed a fog-based model for driving rule monitoring services. Such systems can easily be installed in traffic lights by means of a LiFi communication system to convey information in a smart city environment, where the traffic light communicates to all vehicles in the queue informing them about their individual average speed or driving infractions that happened in the previous streets, for example. However, the LiFi system is not limited to traffic information, since, as considered in Smart Cities applications [2], different types of information regarding the quality of life and city management can be conveyed to the passing vehicles. Building on this, we propose to use the already implemented infrastructure of light in traffic light and cars to convey this information in an efficient manner (since lights have to be used in vehicular systems), that does not use already crowded radio frequencies and provides a fast and reliable data link between cars using LiFi technology.

In this work, we develop a mathematical analysis to study the performance of a LiFi system installed in traffic lights that disseminates information regarding the city using the lights of the vehicles as shown in Figure 1. Building on this, the traffic light acts as the server where information is first

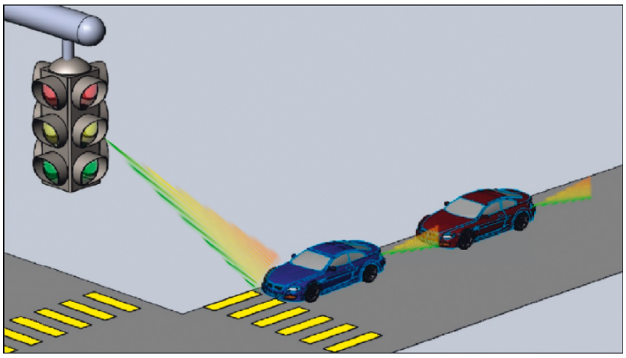

Figure 1: LiFi communication system in traffic lights.

disseminated to vehicles in the city. The first car waiting in the red light receives the packet, and it conveys this information to the car right behind it using its tail lights and the front light of the next car. This procedure is repeated until it reaches the last car in the queue or cars start moving forward. At this point, we assume that vehicles move away from each other and the communication link is lost. This may not be the case in a practical scenario where communication links may still be functional even if vehicles are moving. However, we study the worst case scenario where vehicles can only communicate among them when they are not moving. The relevance of the proposed system is in the implementation of the Internet of Vehicles as studied in [3], where autonomous vehicles cooperate to maintain a smooth traffic flow on roads.

To this end, we assume that vehicles arrive and remain in the vehicle crossing where the traffic light is installed, a random, exponentially distributed time. Indeed, even if the time of the red and green lights is constant, the time that vehicles remain waiting in the intersection is, in many cases, random. This is due to the fact that drivers do not react instantaneously to the red-to-green switching, or they are at times distracted or in a hurry or even because pedestrians are crossing, among other reasons. Hence, we believe that modeling the dwelling time in the system (vehicular crossing) represents a good approximation to a practical system. Furthermore, the use of the exponential distribution corresponds to a first attempt at studying these type of systems. With this in mind, we also consider a simple crossing where no left/right turns are allowed and no U-turns are possible. As such, vehicles can only continue their original direction.

LiFi technology has been studied in the context of vehicular communication systems [4]. For instance, in [5], a cost-effective and inexpensive mechanism for the vehicle-to-vehicle (V2V) communication system using light is proposed. However, only two scenarios were considered. Namely, when a moving car is breaking and it alerts the vehicle behind it to be aware of such speed variation and when a high-speed vehicle approaches a junction it alerts other vehicles that may not detect it. Additionally, in [6], a vehicular communication system based on LiFi is proposed to communicate cars using the front and tail lights to improve road safety and traffic management by storing any infractions from cars, such as maximum speed violation, and sending this information to a central management system and taking legal actions in the future [7]. Conversely, we focus on a more general communication system, where security is an important objective, but other types of information can be conveyed, such as parking spaces, cultural events, weather conditions (rain, fog, or ice) that can affect driving conditions, and others. Another difference is that we focus our study on static or semistatic conditions where cars are waiting for the green light in a traffic light. As such, we study both V2V and V2I (vehicle-to-infrastructure) architectures since cars can communicate with the traffic light. Also, in [2], the authors propose a communication system using LiFi to communicate traffic lights and vehicles. The aim is to optimize traffic flow in the city and avoid car accidents by obtaining the most suitable routes and send alert signals when sudden speed changes are detected. However, the data transmission capacity is not evaluated nor the data throughput of the system.

To the best of our knowledge, this is the first paper that attempts to study the relevance and potential use of the LiFi technology in both V2V and V2I communication systems, clearly showing the benefits and limitations of data dissemination making use of bandwidth outside the already crowded radio frequencies. This is done by deriving the theoretical throughput in different environments. The main contributions of the paper are as follows:

(i) We study the benefits and limitations of an information dissemination system for V2I and V2V downlink communication systems where a LiFi server is installed in traffic lights and vehicles aligned waiting in the queue transmit relevant information

(ii) A mathematical model based on a continuous-time Markov chain (CTMC) is used to evaluate the performance of the LiFi system

(iii) Different scenarios are proposed to evaluate the system performance in terms of average throughput for different packet sizes, data rates, average times that the traffic light is in red/green, and traffic conditions

(iv) Based on the numerical evaluation of the proposed system, we give clear guidelines for the system parameter selection in order to offer adequate throughput in terms of the number of vehicles that can download the data in the junction.

The rest of the paper is organized as follows: Section 2 describes in detail the LiFi system's characteristics that can be used in such communication system. Then, in Section 3, we provide the main assumptions and describe the system that is mathematically modeled in Section 4. We conclude the paper with relevant numerical results presented in Section 5 and Conclusions.

## 2. System Model

The proposed environment considers a vehicle-to-infrastructure (V2I) system where traffic lights transmit information regarding city driving conditions, to avoid traffic jams or roads in construction, for example. Also, diverse information can be downloaded to the vehicles, including weather, pollution, and general information like relevant news and even commercial sales in specific points of the city. As such, some data can be consumed by vehicles while other data can be passed to users and commuters. In the case of autonomous vehicles, this data exchange is of great importance to enhancing the driving conditions by changing routes and adapting speed, effectively reducing commute times in big cities. Such data are proposed to be transmitted using LiFi by taking advantage of light infrastructure already installed throughout cities and in all sort of vehicles, including public transportation buses. This V2I communication is intended for the data transmission from the traffic light to the first car in the queue waiting during the red light. Afterward, this first car conveys this information to the rear car using the tail lights and the front lights, respectively. And this procedure is repeated until the last car in the queue is reached or until the red light turns to the green light and vehicles begin to move, and the car lights are no longer aligned, hindering the communication process. As such, we assume an error-free communication due to an accurate alignment of the transmitters and receivers as well as the close distance among vehicles only when they are not moving during the red light. When cars begin to move, we assume that communication can no longer be established since the distance between vehicles is no longer constant and small, due to the natural movement of the cars moving in the roads, and a good alignment is no longer possible all the time. However, we leave the open issue of a LiFi communication system when vehicles are moving for future research works.

Building on this, the proposed system is presented in Figure 2. It can be seen that the main parameters in the model are as follows:

(i) $Q_1$: vehicular queue for vehicles in direction A. Note that the number of cars in this direction is both the cars coming from the left to the traffic light as well as users arriving from the right.

(ii) $Q_2$: vehicular queue for vehicles in direction B. Note that the number of cars in this direction is both the cars coming from the top to the traffic light as well as users arriving from the bottom.

(iii) $s$: state of the traffic light.

The state of the traffic light, $S$, can be described as follows: when vehicles in direction A have the red light, then cars in direction B are moving since they have the green light. Hence, only vehicles in direction A can communicate through the LiFi system. Conversely, when cars in direction A have the green light, then only cars in direction B can communicate. As such, variable $S$ can be described as a binary variable as follows:

$$S = \begin{cases} 1, & \text{direction A (B) has the green (red) light}, \\ 0, & \text{direction A (B) has the red (green) light}. \end{cases} \quad (1)$$

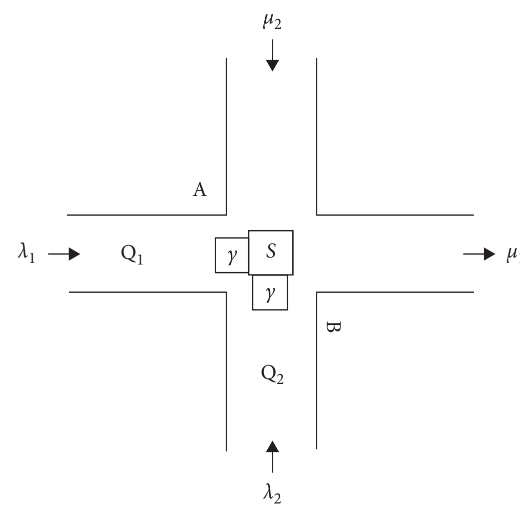


Figure 2: General system model.

For simplicity, we assume that all processes involved in the crossing are Poisson processes. This is a major assumption in the work. However, as a first attempt to study the performance of the system, we believe that it provides an approximation to the throughput of LiFi communications in traffic lights. In future works, we will look into more accurate models in such systems.

Building from this, interarrival times of vehicles in both direction A and B are assumed to be random variables, exponentially distributed with rates $\lambda_1$ and $\lambda_2$, respectively. Also, average waiting times of vehicles in the crossing are assumed to be random variables, exponentially distributed with mean $(1/\mu_1)$ for direction A and $(1/\mu_1)$ for direction B. Finally, traffic light remains in state $s = 0, 1$, an exponentially distributed random time with rate $\gamma$.

Note that we model a crossing where vehicles in each direction do not interfere on the flowing of cars in the other direction. As such, we do not consider a crossing where wide turns nor U-turns are allowed. Finally, a single lane is considered. However, the results derived in this work can be easily extended to multiple lanes if we assume that only cars in the same lane can communicate among them, i.e., cars in a particular lane cannot communicate with cars in another lane.

## 3. LiFi System

Light-fidelity is a technology that uses electromagnetic waves in the visible range (380 nm to 750 nm approximately [8]). Therefore, LiFi presents, as its name implies, a broad fidelity in light communications, since it provides high transmission data rates (10 Gbps [9]), free bandwidth, high level of security, and high propagation capabilities with respect to other types of waves, because transmissions though water, terrestrial surface, and even in outer space are possible. LiFi has been cataloged as a VLC (visible light communication) technology, because it is a light transmission technology. The main differentiator of LiFi with respect to the other VLC technologies lies in the fact that the previous VLC technologies have been conceived mainly with PPP (point to point) communication, that is, as a substitute for a cable, while LiFi has the characteristic to be a complete network system with bidirectional and multiuser communication.

LiFi uses light emitting diodes (LED's) to make full connections in wireless network systems. Each LED in a LiFi system acts as an access point, and due to the size (order of millimeters) of the AP's, their networks are called attocell networks, unlike WiFi femtocell networks. These improvements in the attocell networks provide the necessary infrastructure for the IoT technologies and contribute to the fifth generation 5G [10] of cellular systems. LiFi arises from a worldwide need, the scarcity of spectrum bands available for wireless-fidelity (WiFi) technology.

The use of LiFi systems in indoor applications could represent the largest field of action of this technology in the coming decades since it could become a solution for massive deployment of nodes, making use of bandwidth-efficient [11] and secure connections [12] in the Internet of Things. As a result of many experimentation in modulation [13], color and power, the use of LiFi has been envisioned to be used in urban, marine, and in outer space environments. To this date, the use of LiFi is mainly focused for indoor environments where illumination conditions are rather stable, where nodes have low or no mobility and there are few obstacles between transmitters and receivers. However, current efforts on LiFi allows the use of this technology in outdoor environments like the one shown in Figure 3 where we depict efficient-bandwidth use scenarios in future communications systems.

In this case, the LiFi system is composed of the following components as shown in Figure 4:

(i) Data source: provides the data that are transmitted through the system, and it is expected that the speeds reach the order of gigabits per second

(ii) LED transmitter system

(iii) LED driver: this block is responsible for supplying the required energy at the traffic lights

(iv) Traffic lights: on this part of the system, the information previously treated (modulated, coded, amplified, etc.) will be emitted through the traffic lights; according to the current data on long-range LEDs [14], it is estimated that the power at which the signal from the traffic lights will be irradiated will be a couple of tens of Watts, reaching a distance of up to 30 meters in a cone of coverage of 10 meters, while the headlights of the vehicles could transmit data in a range of up to 300 meters in a cone of coverage of 20 meters

(v) Light: this will be the transmission medium through which the information will be sent, and it contains a bandwidth of the order of the tera-hertz

(vi) LED receiver system

(vii) Light sensor: this accessory will be added to the vehicles in order to obtain the data transmitted through the light

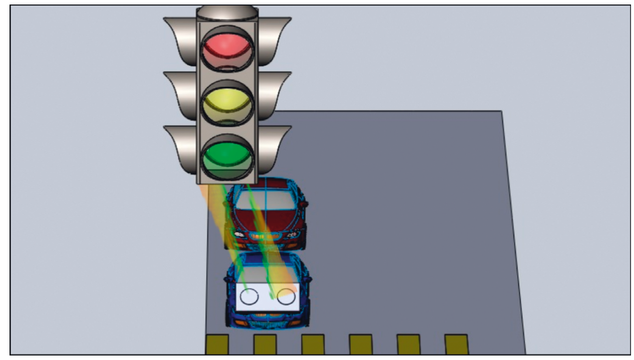

Figure 3: LiFi communication system in traffic lights.

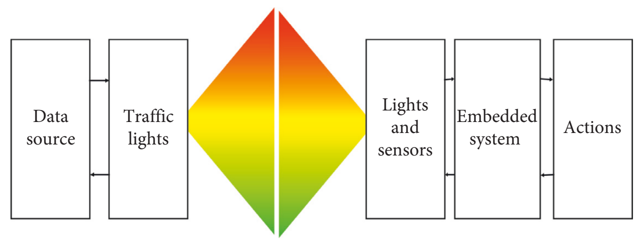


Figure 4: General LiFi communication system.

(viii) Embedded system: once the signal is obtained, this block will perform the necessary treatment on the signal to recover the information

(ix) Actions: the information acquired will allow vehicles to carry out actions based on their programming, from displaying information on a screen to stopping the vehicle completely

In more detail, the basic circuit developed for the light reception is shown in Figure 5, where we can see that it has four main components: photodiode, current-to-voltage converter, voltage comparator, and microprocessor. (In this case, we use an ARM cortex M4 for our experiments.) The photodiode acts as a light source, gathering the light from the transmitter and varying the current intensity according to the excitement at the input. Then, the current transits through the first operational amplifier which acts as a current-to-voltage converter and also provides a certain gain to the output signal as depicted in Figure 6. This output signal enters the microcontroller ARM in order to process the pulses, and information can be extracted. For the transmission circuit shown in Figure 7, the microcontroller sends the digital pulses through the first amplifier, acting as an impedance coupler in order to excite the terminal gate of the MOSFET device. Then, the type N MOSFET turns ON and OFF the LEDs depending on the value of the pulse at its input. We performed our experiments using the Sisoft (a Mexican startup) LUMUX/Dongle, commercial devices, used for LiFi applications for indoor usage as shown in Figure 8 with the following characteristics: voltage operation 127–240 volts, consumption power: 20 watts, temperature color: 5000 K, power factor: 0.9, CRI: 85, aperture angle: 110° per side, wavelength: 850 nm, communication distance:

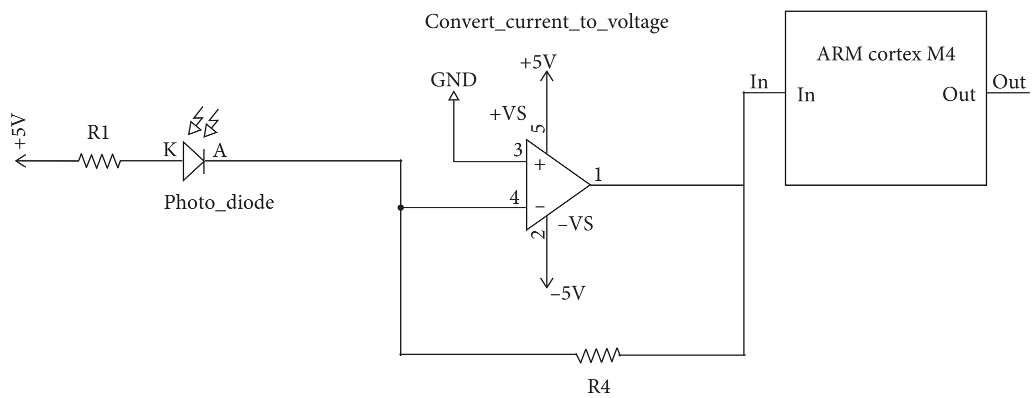


Figure 5: Reception circuit.

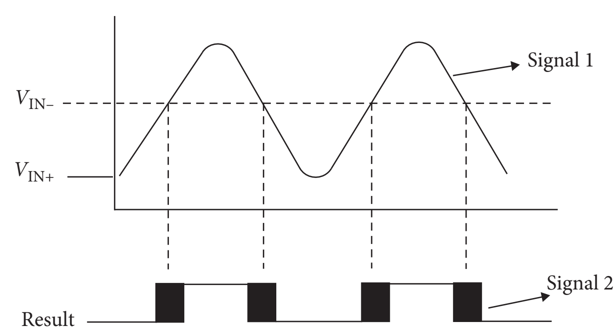


Figure 6: Output signal at the current-to-voltage converter.

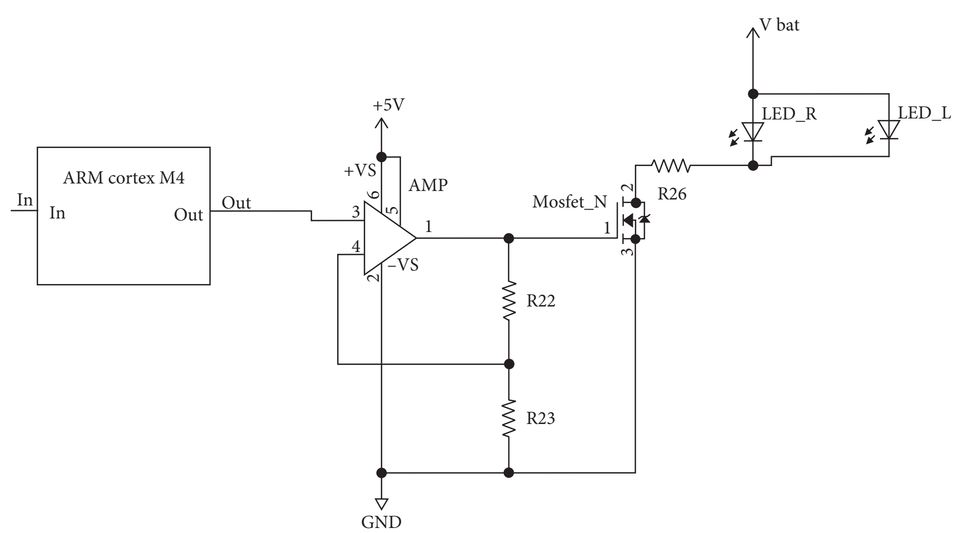


Figure 7: Transmission circuit.

2.2 m, data rate: 8 mbps both uplink and downlink, radiation intensity: 500 mW/sr, photo-current: 500 nA, operation temperature: −40 to 85 centigrade degrees, and modulation: OOK.

For the outdoor application proposed in this work, the devices that are implemented in the vehicles will obtain the energy from an angle and will reflect a portion of it, thus complying with the law of Snell [15]. Therefore, there will be a critical angle in which the reflection of the signal will be a total reflection (Figure 9).

Tables 1 and 2 show relevant characteristics provided by the protocol IEEE 802.15.7 [16].

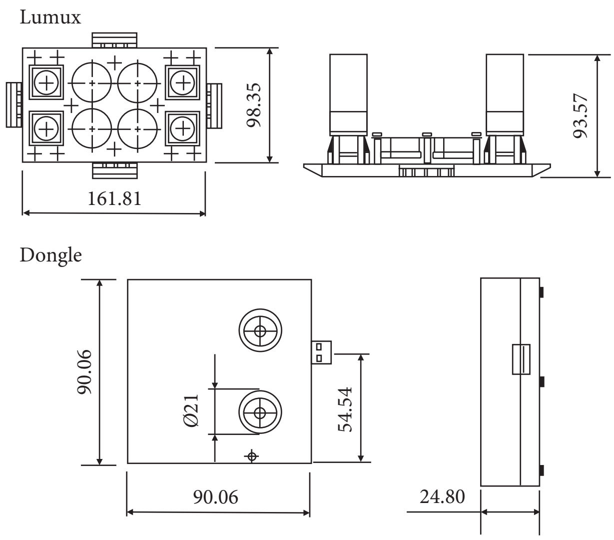


Figure 8: LUMUX/Dongle devices.

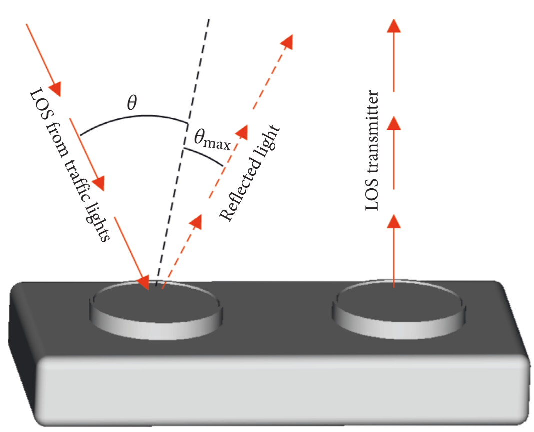


Figure 9: LiFi car adaptation in a VANET environment.

Table 1: Device classification according to the IEEE 802.15.7 standard.

| | Infrastructure | Mobile | Vehicle |
|---|---|---|---|
| Fixed coordinator | Yes | No | No |
| Power supply | Ample | Limited | Moderate |
| Data rates | High/low | High | Low |
| Range | Short/long | Short | Long |
| Shape factor | Unconstrained | Constrained | Unconstrained |
| Light source | Intense | Weak | Intense |
| Physical mobility | No | Yes | Yes |

## 4. Mathematical Analysis

Based on the system description in the previous section, we develop a continuous-time Markov chain (CTMC) with valid state space $(\Omega : Q_1, Q_2, s \,|\, 0 \le Q_1, 0 \le Q_2,\ s = 0, 1)$. The system is assumed to start at state (0, 0, 0) and evolves accordingly. At an arbitrary state $(Q_1, Q_2, s)$, $Q_1 > 0$, $Q_2 > 0$, the valid state transitions are as follows and are depicted in Figure 10:

(i) To state $(Q_1 + 1, Q_2, s)$ with rate $\lambda_1$: in this case, a new vehicle arrives at direction A of the crossing.

(ii) To state $(Q_1, Q_2 + 1, s)$ with rate $\lambda_2$: in this case, a new vehicle arrives at direction B of the crossing.

(iii) To state $(Q_1 - 1, Q_2, s)$ with rate $s * \mu_1$: in this case, a vehicle driving in direction A of the crossing successfully leaves the intersection.

(iv) To state $(Q_1, Q_2 - 1, s)$ with rate $(1 - s) * \mu_2$: in this case, a vehicle driving in direction B of the crossing successfully leaves the intersection.

(v) To state $(Q_1, Q_2, \bar{s})$ with rate $\gamma$: in this case, the traffic light switches to 1 (0) in case that the current state of $s$ is 0 (1). In other words, if the traffic light is green (red) in direction A (B), then it goes to red (green) in direction A (B). Conversely, if the traffic light is red (green) in direction A (B), then it goes to green (red) in direction A (B).

It is important to note that the proposed Markov chain corresponds to a M/M/1 switched Markov chain, where transitions to some states are not allowed depending on the current state. Specifically, note that when vehicles in direction A have the green light, $s = 1$, vehicles in direction B cannot leave the intersection since they have the red light (and transition rates to state $Q_2 - 1$ is $(1 - s) * \mu_2 = 0$). On the other hand, when vehicles in direction A have the red light, $s = 0$, transitions to state $Q_1 - 1$ are not possible (this rate is $s * \mu_1 = 0$).

In this model, particular cases have to be considered. For instance, at state $(Q_1, Q_2, s)$, $Q_1 = 0$, $Q_2 > 0$, the valid state transitions are as follows and are depicted in Figure 11:

(i) To state $(Q_1 + 1, Q_2, s)$ with rate $\lambda_1$: in this case, a new vehicle arrives at direction A of the crossing

(ii) To state $(Q_1, Q_2 + 1, s)$ with rate $\lambda_2$: in this case, a new vehicle arrives at direction B of the crossing

(iii) To state $(Q_1, Q_2 - 1, s)$ with rate $(1 - s) * \mu_2$: in this case, a vehicle driving in direction B of the crossing successfully leaves the intersection

(iv) To state $(Q_1, Q_2, \bar{s})$ with rate $\gamma$: in this case, the traffic light switches to 1 in case that the current state of $s$ is 0

The rest of these special cases can be easily inferred, i.e., cases where $Q_1 > 0$, $Q_2 = 0$ and $Q_1 = 0$, $Q_2 = 0$. Building on this, Figure 12 shows the evolution of the proposed system. The Markov chain is solved in order to find the steady state probabilities, $\overline{\Pi} = [\pi_{(0,0,0)}, \pi_{(1,0,0)}, \pi_{(2,0,0)}, \ldots, \pi_{(0,1,0)}, \pi_{(0,2,0)}, \ldots]$. Since this chain represents a conventional birth/death process, the proposed system has steady state probabilities, $\Pi$, when $\rho = \lambda/\mu < 1$. That is, the traffic load (relation between the arrival and departure rates) is lower than one. The steady state probabilities are found by equalizing the transition rates as follows:

Table 2: Mode PHY I.

| Modulation | Code RLL | Frequency optic (kHz) | FEC RS | FEC CC | Data flow (kb/s) |
|---|---|---|---|---|---|
| OOK | Manchester | 200 | (15,7) | 1/4 | 11.67 |
| | | | (15,11) | 1/3 | 24.44 |
| | | | (15,11) | 2/3 | 18.89 |
| | | | (15,11) | None | 73.3 |
| | | | None | None | 100 |
| VPPM | 4B6B | 400 | (15,2) | None | 35.56 |
| | | | (15,4) | None | 71.11 |
| | | | (15,7) | None | 124.4 |
| | | | None | None | 266.6 |

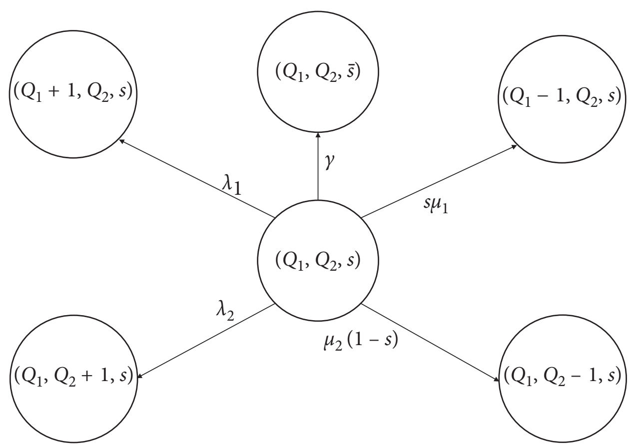


Figure 10: Proposed Markov chain to model the vehicle intersection.

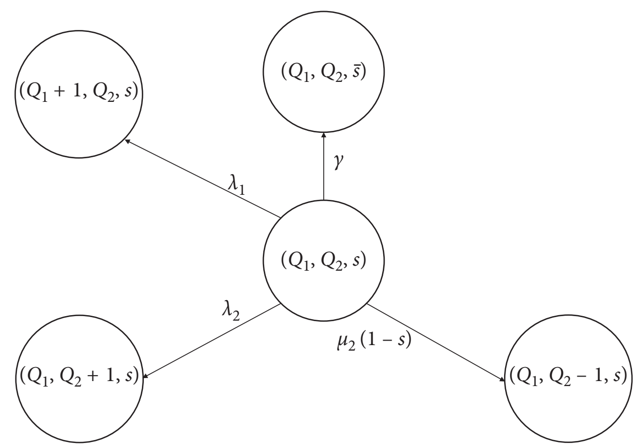


Figure 11: Valid transitions in the Markov chain when $Q_1 = 0$.

$$\begin{aligned}
(\lambda_1+\lambda_2+\gamma)\pi_{(0,0,0)} &= \mu_1\pi_{(1,0,0)}+\mu_2\pi_{(0,1,0)}+\gamma\pi_{(0,0,1)},\\
(\lambda_1+\lambda_2+\mu_2+\gamma)\pi_{(0,1,0)} &= \lambda_2\pi_{(0,0,0)}+\cdots+\gamma\pi_{(0,1,1)},\\
&\vdots\\
(\lambda_1+\lambda_2+\mu_1+\mu_2+\gamma)\pi_{(15,7,1)} &= \lambda_1\pi_{(14,7,1)}+\cdots+\gamma\pi_{(15,7,0)},\\
&\vdots\\
(\lambda_1+\lambda_2+\mu_1+\mu_2+\gamma)\pi_{(q_1,q_2,s)} &= \lambda_1\pi_{(q_1-1,q_2,s)}+\cdots+\gamma\pi_{(q_1,q_2,\bar{s})}.
\end{aligned} \tag{2}$$

Specifically, the aforementioned system is numerically solved, as depicted in Algorithm 1, and we now describe it in detail:

(i) The process starts introducing all variables as the input and output rates, the exchange rates on the semaphore, and the number of iterations in the system (depicted as $\mu_1, \mu_2, \lambda_1, \lambda_2$, and $\gamma$ and transitions, respectively) in order to obtain the average length and delay in the queue

(ii) Once the input variables are introduced, we start the simulation time, the iteration counter, and the initial values for the queues and the semaphore (defined as $t_{\text{sim}}, \text{count}, Q_1, Q_2$, and $s$, respectively) assigning 0 at the initial value

(iii) After creating the initial state as $Q_1, Q_2$, and $s$, it is added on the queue (depicted by instruction List. addState $(Q_1, Q_2, s)$)

(iv) Then, the process below is performed while the iteration counter is less than the number of iterations required to obtain statistically valid steady state distributions of the Markov chain:

(1) As a first step, exponential times are created by the input and output rates at the crossing and the exchange rate at the traffic light, an dthese exponential times are saved in a vector for later comparison (*expTime* [5])

(2) Once the values were saved, the minimum time ($t_{\min}$) in the vector and its position (*position*) are determined with *tminPosition*(expTime [5])

(3) Next, the state ID is obtained in order to corroborate its existence in the queue (*currentIID*)

(4) Once the existence of the state in the queue has been checked, there are two possibilities: if the state ID does not exist, it is created in the queue and the minimum time of the current iteration ($t_{\min}$) is added; on the contrary, if the state previously existed, the value of the previous time in which the simulation was kept in that state ($t_{\text{previous}}$) is taken and the minimum time of the current iteration is added

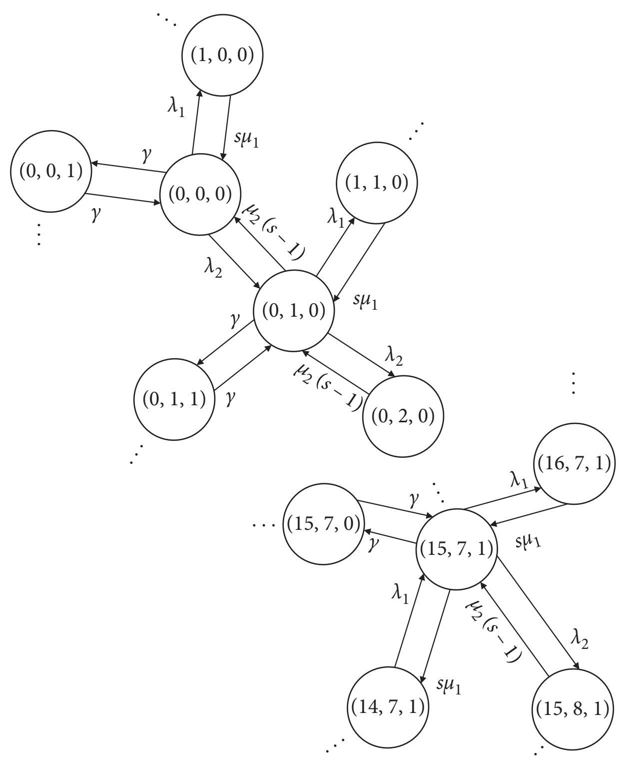


FIGURE 12: Time evolution of the Markov chain to model the vehicle intersection.

(5) When the previous condition is finished, the minimum time of the iteration is added to the simulation time ($t_{\text{sim}}$)
(6) The value corresponding to the position of the minimum time of the current iteration will define the state of the next iteration, increasing or decreasing the value of the queues at the cross roads or inverting the value of the traffic light ($Q_1+1$, $Q_2+1$, $Q_1-1$, $Q_2-1$, or *invertTrafficlight* ($S$))

(v) When the number of iterations is met, the steady probability is calculated and added to each state in the queue (*probabilities* (*List*, $t_{\text{sim}}$))
(vi) Finally, the queue length and delay are calculated by Little's theorem (*Little theorem* (*List*, $\lambda_1$, $\lambda_2$))

Now, the average number of vehicles in the crossing, $\overline{L}$, is found as follows:

$$\overline{L} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{1}(i+j)\Pi_{i,j,k}. \quad (3)$$

Finally, using Little's Theorem, the average time spent in the vehicle junction, $\overline{D}$ can be found as

$$D = \frac{\overline{L}}{\lambda_1+\lambda_2}. \quad (4)$$

Note that we are only considering the average number of vehicles in the crossing as well as the average time spent in such junction. Hence, we are not considering the instantaneous number of vehicles. Therefore, the proposed model does not consider the cases where some vehicles successfully leave the crossing (when the traffic light in their direction is green) while some vehicles cannot leave the crossing, remaining there for multiple changes of the traffic light.

We now calculate the average number of packets that can be transmitted while waiting in the red light, which depends on the packet size, data rate, and average waiting time. We focus on smart city applications, where general information is sent to all drivers concerning driving conditions, traffic reports, and security issues (like roadblocks, accidents, and public event, among others) and even information for the vehicles in autonomous car scenarios. As such, all vehicles in the crossing are interested in downloading the data from the server (in this case, the traffic light), even if some data are passed on to the drivers while some data are consumed by the own vehicle.

To this end, we first calculate the time required for a packet to be transmitted from the traffic light to the first car in the queue (from the red light to the front lights of the vehicle) which we assume to be the same time required to transmit the packet from the tail lights of this car to the front light of the vehicle just behind it, and so on and so forth. This packet transmission can be calculated as

$$t_{\text{t}} = \frac{T_{\text{p}}}{V_{\text{t}}}\ (\text{s}), \quad (5)$$

where $t_{\text{t}}$ is the packet transmission time (s) per vehicle, $V_{\text{t}}$ is the data rate of the LiFi system (b/s), and $T_{\text{p}}$ is the packet size (b). Hence, the time required to transmit the packet to the average number of vehicles waiting in the red light is simply as follows: $t_{\text{t}}^{(\overline{L})} = \overline{L}(t_{\text{t}})$ (s). However, not all cars waiting in the traffic light may be able to download the packet. Indeed, if the time required to transmit the packet to all vehicles waiting in the traffic light is lower than the average waiting time ($t_{\text{t}}^{(\overline{L})} \le \overline{D}$), then the packet can go from the traffic light to the last car (in average) in the queue before the green light is switched on and cars begin to move. Conversely, if the average time to convey the packet in the junction is greater than the average waiting time ($t_{\text{t}}^{(\overline{L})} \le \overline{D}$), then the packet cannot reach all vehicles and only a fraction of these cars ($\lfloor \bar{D}/t_{\text{t}}$) receive the packet. As such, the average number of vehicles that receive the packets in a red light, $\overline{\sigma}$, can be expressed as

$$\overline{\sigma} = \begin{cases} \overline{L}, & \text{if } t_{\text{t}}^{(\overline{L})} \le \overline{D}, \\ \left(\dfrac{\overline{D}}{t_{\text{t}}}\right), & \text{if } t_{\text{t}}^{(\overline{L})} > \bar{D}. \end{cases} \quad (6)$$

## 5. Numerical Results

In this section, we present some relevant results of the proposed Markov chain that models the road system. For the results derived in this section, we consider the following

```
Data: μ1, μ2, λ1, λ2, γ, Q1, Q2, S, count, transitions, t_simulation
Result: L̄, D̄
t_simulation = Q1 = Q2 = S = count ⟵ 0
   List.addState (Q1, Q2, S);
while count < transitions do
      expTime [5] ⟵ createTime (μ1, μ2, λ1, λ2, γ, S);
      [t_min, position] ⟵ tminPosition (expTime [5]);
      actualID ⟵ concatenate (Q1, Q2, S);
      if: ID ∃! then
         List.addElement (Q1, Q2, S);
         List.getElement (ID).time ⟵ t_min;
      else
         List.getElement (ID).time ⟵ t_previous + t_min;
      end
      t_simulation = t_simulation + t_min;
      switch position do
         case 0 do
            Q1 ⟵ Q1 + 1
         end
         case 1 do
            Q2 ⟵ Q2 + 1
         end
         case 2 do
            S ⟵ invertTrafficlight (S);
         end
         case 3 do
            Q1 ⟵ Q1 − 1
         end
         case 4 do
            Q2 ⟵ Q2 + 1
         end
         otherwise do
         end
      end
end
probabilities (List, t_simulation);
[L̄, D̄] ⟵ Littletheorem (List, λ1, λ2);
```

Algorithm 1: Little's theorem (average length queue and average delay queue).

parameters: $\mu_1 = \mu_2 = 10$, and the traffic load at the crossing given by $\rho = (\lambda/\mu)$. Also, the reference unit of time of the system rates, $\lambda, \gamma$, and $\mu$, is in minutes. Furthermore, we consider that the traffic light remains in the same state (either red or green in either direction) for an average time $(1/\gamma)$ in the range of 10.9 sec (1/5.5 minutes) to 1 minute, in order to cover a wide range of possible scenarios in urban environments.

First, we focus on the average number of cars waiting in the traffic light. Figure 13 depicts the average number of vehicles, $\overline{L}$, waiting in the traffic light for different values of traffic loads in both directions and switch rate of the traffic light. As expected, $\overline{L}$ increases as traffic load increases. A less evident result is that, as the average time in red (and green) increases ($\gamma$ decreases), the average number of vehicles also increases for any value of the traffic load. This is not a straightforward result since as $\gamma$ decreases, the average time that vehicles in a given direction find the traffic light in green increases. Hence, there is less probability of finding vehicles in such direction. However, the cars waiting in the other direction, where the red light is on, remain longer times waiting for the green light, effectively increasing the average number of vehicles in the crossing. This could suggest that selecting high waiting times in traffic lights would be beneficial for the data transmission in the proposed LiFi system (indeed, high values of $\overline{L}$ implies that a high number of vehicles can potentially download the data transmitted by the traffic light, and information relevant to the smart city environment can be efficiently conveyed in these crossings). However, such long waiting times could lead to detrimental traffic jams as vehicles pile up in the crossing. Then, arbitrarily high waiting times cannot be selected. We believe that average times in red (green) of 1 minute in a simple crossing as the one considered in this work (no right/left turning or U-turns are available) can be considered as rather long, causing considerable inconveniences to drivers. Building on this, average times in the red light of less than 30 seconds ($\gamma \geq 2$) would be acceptable. In this range, the difference in

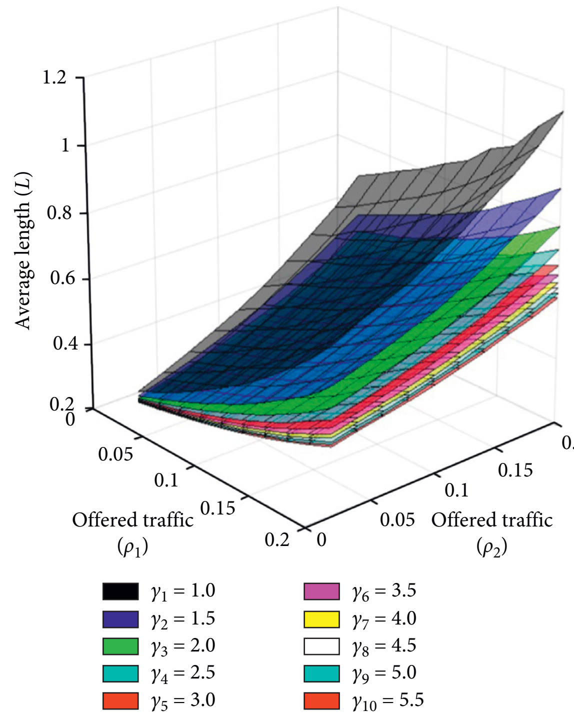


FIGURE 13: Average queue length in the vehicular crossing.

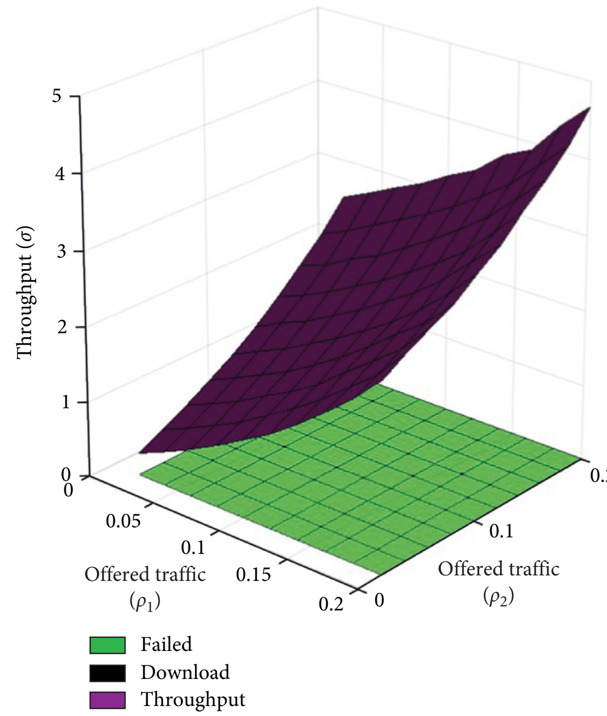


FIGURE 15: Single jumbo frame transmission performance.

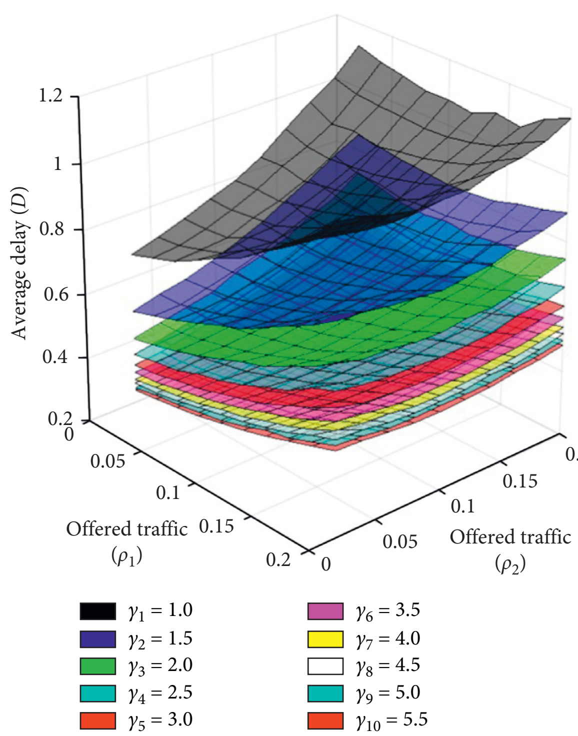


FIGURE 14: Average waiting delay in the vehicular crossing.

the average number of vehicles in the crossing is small for any value of traffic load.

As $\overline{L}$ increases, also the average waiting time, $\overline{D}$, in the crossing increases as shown in Figure 14. As in the case of $\overline{L}$, high values of $\overline{D}$ entails a higher number of vehicles that can potentially download the data disseminated by the traffic light. However, this implies unnecessary waiting times for vehicles hindering the traffic flow in the city.

Now, we focus on the average number of vehicles that successfully download the packet transmitted by the traffic light, $\overline{\sigma}$. To this end, we consider a data rate transmission of $V_t = 11.67$ kb/s with OOK Manchester coding and an optical clock speed of 200 kHz. Also, for these experiments, we consider two packet sizes: Ethernet (1500 bytes) and jumbo frame (9000 bytes). Also a value of $\gamma_1 = 1.0$ is used for the next set of experiments.

First, we consider the case of jumbo frame transmissions since it allows higher information exchange rates. Figures 15 and 16 show the system performance for a single jumbo frame and 8 jumbo frames, respectively. The rationale behind this selection is to consider a wide range of possible applications in the smart city context that goes from small text messages to a short video transmission. In these results, the ideal data download corresponds to the average number of vehicles in the crossing, i.e., the average number of packets that have to be transmitted from the traffic light to the car at the top of the line to the vehicle right behind it and so on. For a single jumbo frame transmission, we can see that the LiFi system is capable of transmitting the data packet to the average number of vehicles waiting in the crossing. As such, the data download surface (in blue) matches exactly the ideal data download and there is no data that are not downloaded (green surface). Note that as the traffic load increases, also the transmission capacity increases since there are more vehicles available in the crossing to pass the information. However, for the case of 8 jumbo frames, we can see that all vehicles are able to

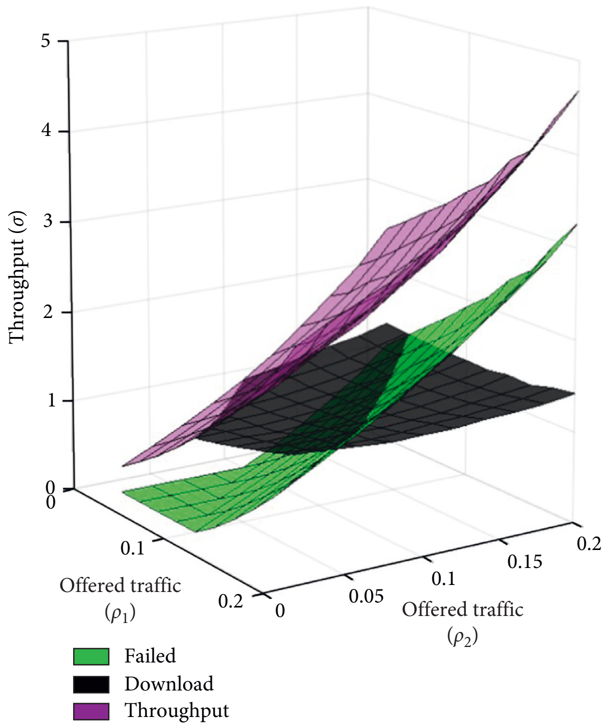


Figure 16: Eight jumbo frames' transmission performance.

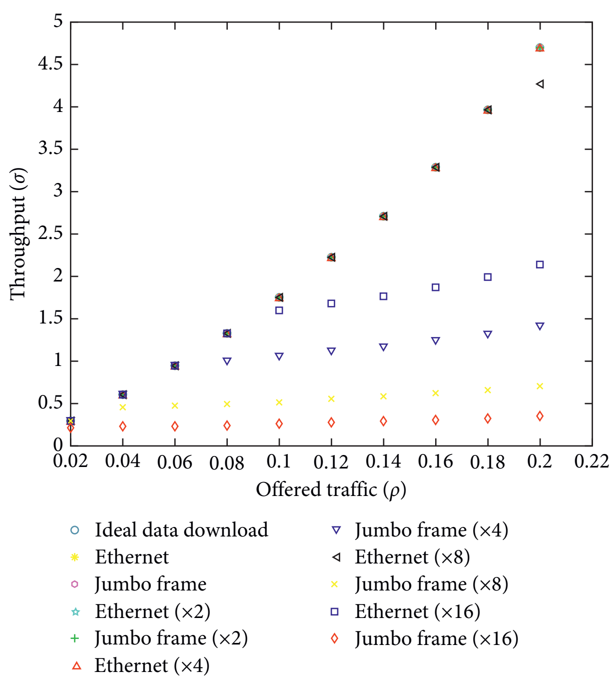


Figure 17: Average number of jumbo frames and Ethernet frames downloaded for OOK, Manchester, 200 kHz, and 24.44 kb/s.

download the city data only for traffic loads lower than 0.1 erlangs. For traffic loads higher than 0.1 erlangs, there are packets that are not conveyed to some vehicles in the crossing (green surface) while the number of packets effectively transmitted remain constant since no more cars can effectively download the information.

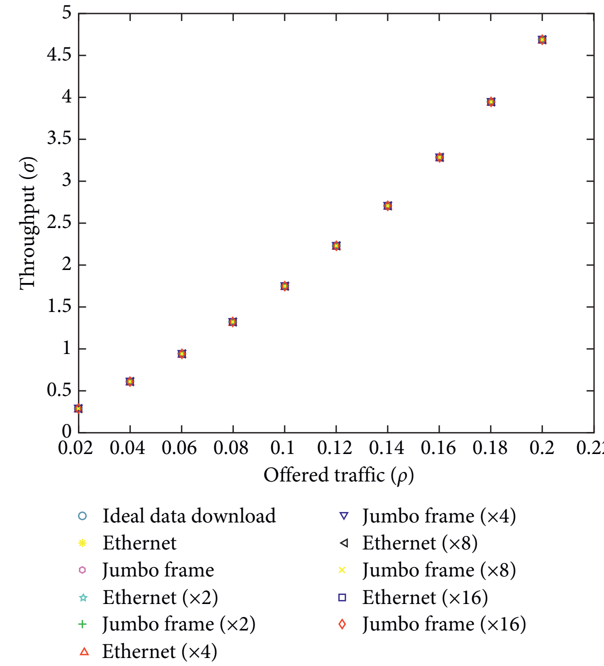


Figure 18: Average number of jumbo frames and Ethernet frames downloaded for VPP, 4B6B, 400 kHz, and 266.6 kb/s.

We believe that, based on the previous results, this work presents clear guidelines for the selection on packet size and data adequate for such LiFi system, based on the particular traffic conditions of the city, i.e., the traffic load in the vehicular crossing. To see this in more detail, Figures 17 and 18 depict the average number of vehicles that download the data packet for different packet sizes and data rates. In these figures, both directions in the crossing have the same offered load, $\rho$. Specifically, Figure 17 considers the case of a LiFi system with OOK modulation and Manchester codification and clock speed of 200 kHz that provides 24.44 Kbps. In this system, it can be seen that up to 16 Ethernet frames can be effectively transmitted to all vehicles in the crossing, except for traffic loads higher than 0.18 erlangs. Conversely, up to 2 jumbo frames can be conveyed to all vehicles in the crossing for any traffic load. When 4 jumbo frames are transmitted, less than 3 vehicles are able to download the information, and in the case of 16 jumbo frames, only one vehicle is able to download the packet. By increasing the data rate to 266.6 Kbps, all 16 Ethernet and jumbo frames can be effectively transmitted to the average number of vehicles in the crossing as shown in Figure 18, where VPP modulation, 4B6B codification, and speed clock of 400 kHz are used.

## 6. Conclusion

In this work, a mathematical analysis based on a CTMC is presented to model a vehicular crossing where traffic lights are used to convey information to vehicles passing by and waiting for the green light. A LiFi system is proposed to be used in order to make a bandwidth-efficient system by making use of different frequencies than the ones used in cellular, WiFi, and Bluetooth which are overcrowded and will experience even more traffic when 5G communication systems get deployed. Conversely, the proposed LiFi system takes advantage of visible light already used in traffic lights and front and tail lights in vehicles.

We prove the effectiveness of such systems for applications of smart cities, where information regarding traffic conditions, security, cultural, or even emergency events can be effectively conveyed to cars waiting in the red light. This data can be disseminated using small text messages to small video files. Also, we provide clear guidelines for packet sizes and data rates to be used in the LiFi network in order to transmit data to the average number of cars that can be found in the crossing. Hence, the system administrator can use the proposed model to accurately select the system parameters for different crossings based on the traffic load at each strategic traffic light of the city.

In the future work, we propose to model more complex vehicular crossings, where left/right and U-turns are allowed. Also, the case where information can be transmitted from vehicle to vehicle in motion and not only during the red light can be studied, considering the average times that cars are connected in such dynamic cases. Also, the exponential assumption can be tested and changed if it does not hold in practical cases.

## Data Availability

All data used in the experiments can be made available if required, and they were obtained analytically by solving the proposed Markov chain.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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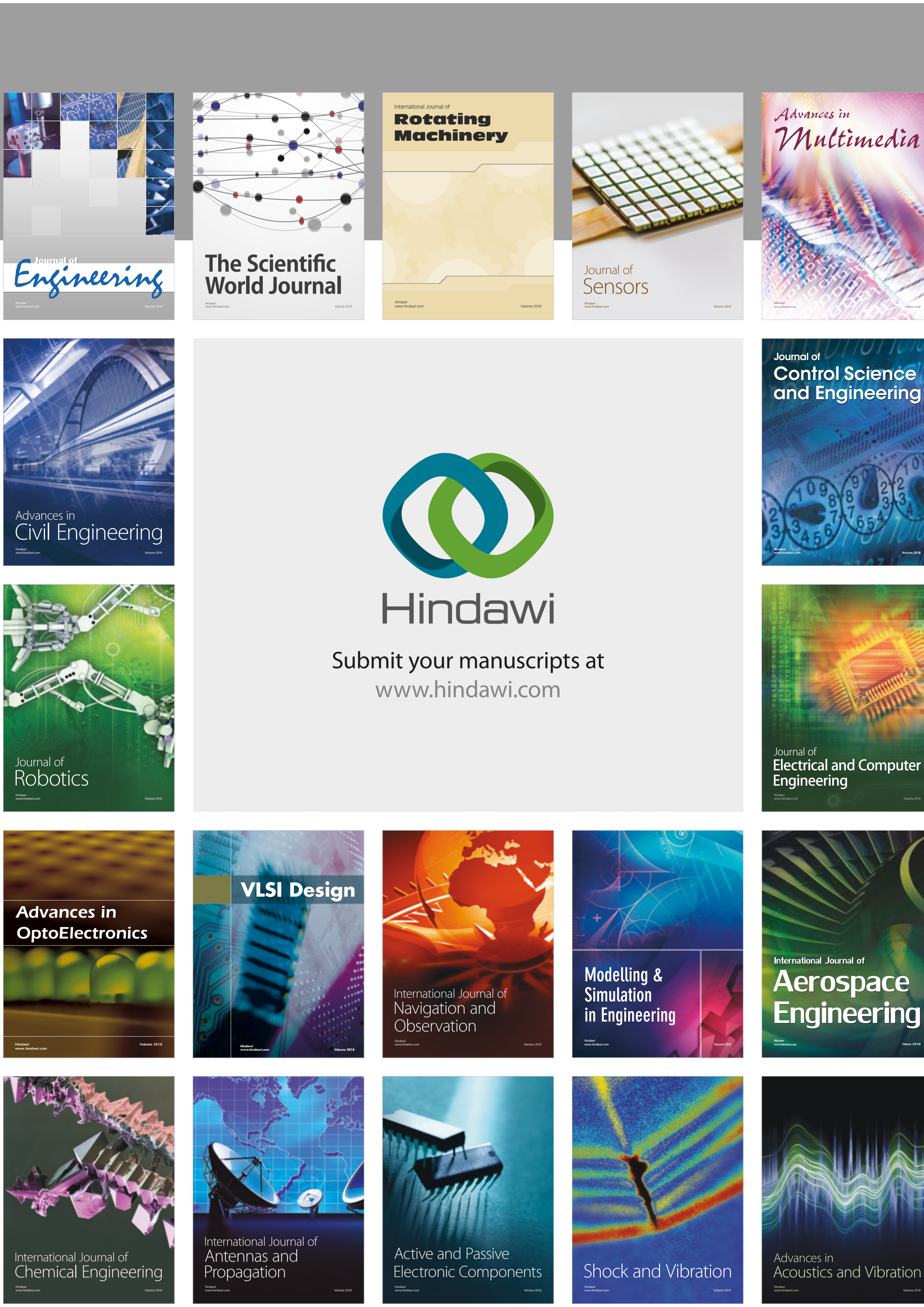
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