DIRECTORIO DE PROFESORES DEL CURSO DE OBRAS MARITIMAS.- "HIDRAULICA MARITIMA Y DE ESTUARIOS".- DEL 11 DE MARZO AL 3 DE JUNIO. 1985.

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UNAM.
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EVALUACION DEL PERSONAL DOCENTE

CURSO:

FECHA:

CONFERENCISTA	DOMINIO DEL TEMA	EFICIENCIA EN EL USO DE AYUDAS AUDIOVISUALES	MANTENIMIENTO DEL INTERES. (COMUNICACION CON LOS ASISTENTES, AMENIDAD, FACILIDAD DE EXPRESION).	PUNTUALIDAD	
1 ING. OSCAR FUENTES MARILES					
2.					
3.					
4.					
5.					
6.					
7.					
9.					

EVALUACION DE LA ENSEÑANZA

SU EVALUACION SINCERA NOS AYUDARA A MEJORAR LOS PROGRAMAS POSTERIORES QUE DISEÑAREMOS PARA USTED.

TEMA	ORGANIZACION Y DESARROLLO DEL TEMA	GRADO DE PROFUNDIDAD LOGRADO EN EL TEMA	GRADO DE ACTUALIZACION LOGRADO EN EL TEMA	UTILIDAD PRACTICA DEL TEMA	
ONDAS					
TRANSFORMACIONES DE LAS ONDAS					
ONDAS OCEANICAS					
PREDICCION DEL OLEAJE					
EFECTO DEL OLEAJE SOBRE ESTRUCTURAS MARITIMAS.					
PROCESOS COSTEROS					
ESTUARIOS					
ONDAS DE LARGO PERIODO					

EVALUACION DEL CURSO

	CONCEPTO	EVALUACION
1.	APLICACION INMEDIATA DE LOS CONCEPTOS EXPUESTOS	
2.	CLARIDAD CON QUE SE EXPUSIERON LOS TEMAS	
3.	GRADO DE ACTUALIZACION LOGRADO CON EL CURSO	
4.	CUMPLIMIENTO DE LOS OBJETIVOS DEL CURSO	
5.	CONTINUIDAD EN LOS TEMAS DEL CURSO	
6.	CALIDAD DE LAS NOTAS DEL CURSO	
7.	GRADO DE MOTIVACION LOGRADO CON EL CURSO	

ESCALA DE EVALUACION DE 1 A 10

1. ¿Qué le pareció el ambiente en la División de Educación Continua?

MUY AGRADABLE	AGRADABLE	DESAGRADABLE

2. Medio de comunicación por el que se enteró del curso:

PERIODICO EXCELSIOR ANUNCIO TITULADO DIVISION DE EDUCACION CONTINUA	PERIODICO NOVEDADES ANUNCIO TITULADO DIVISION DE EDUCACION CONTINUA	FOLLETO DEL CURSO

CARTEL MENSUAL	RADIO UNIVERSIDAD	COMUNICACION CARTA, TELEFONO, VERBAL, ETC.

REVISTAS TECNICAS	FOLLETO ANUAL	CARTELERA UNAM "LOS UNIVERSITARIOS HOY"	GACETA UNAM

3. Medio de transporte utilizado para venir al Palacio de Minería:

AUTOMOVIL PARTICULAR	METRO	OTRO MEDIO

4. ¿Qué cambios haría usted en el programa para tratar de perfeccionar el curso? ______________________________________

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5. ¿Recomendaría el curso a otras personas?

SI	NO

6. ¿Qué cursos le gustaría que ofreciera la División de Educación Continua?

____

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7. La coordinación académica fue:

EXCELENTE	BUENA	REGULAR	MALA

8. Si está interesado en tomar algún curso <u>intensivo</u> ¿Cuál es el horario más conveniente para usted?

LUNES A VIERNES DE 9 A 13 H. Y DE 14 A 18 H. (CON COMIDAS)	LUNES A VIERNES DE 17 A 21 H.	LUNES, MIERCOLES Y VIERNES DE 18 A 21 H.	MARTES Y JUEVES DE 18 A 21 H.

VIERNES DE 17 A 21 H. SABADOS DE 9 A 14 H.	VIERNES DE 17 A 21 H. SABADOS DE 9 A 13 Y DE 14 a 18 H.	O T R O

9. ¿Qué servicios adicionales desearía que tuviese la División de Educación Continua, para los asistentes? ______________________________

____

10. Otras sugerencias:

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____

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DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

CURSO: OBRAS MARITIMAS
"HIDRAULICA MARITIMA Y DE ESTUARIOS"
DEL 11 DE MARZO AL 3 DE JUNIO
MEXICO, D.F.

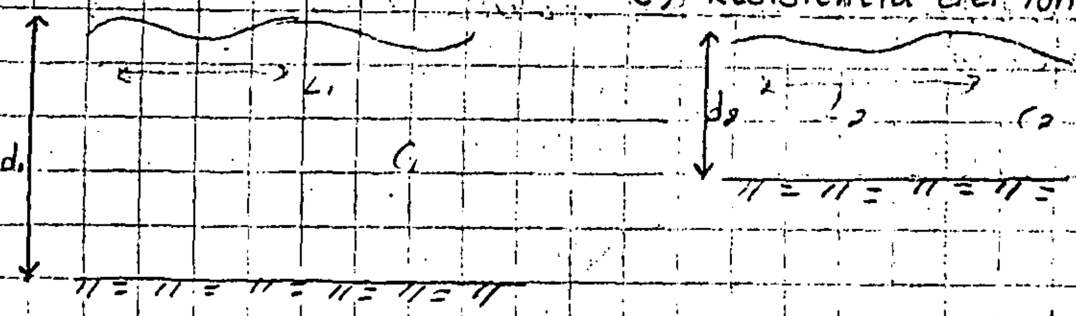
TRANSFORMACIONES DE LAS ONDAS

ING. OSCAR FUENTES MARILES
JUNIO 1985

2. TRANSFORMACIONES DE LAS ONDAS

Modificaciones de las ondas progresivas.

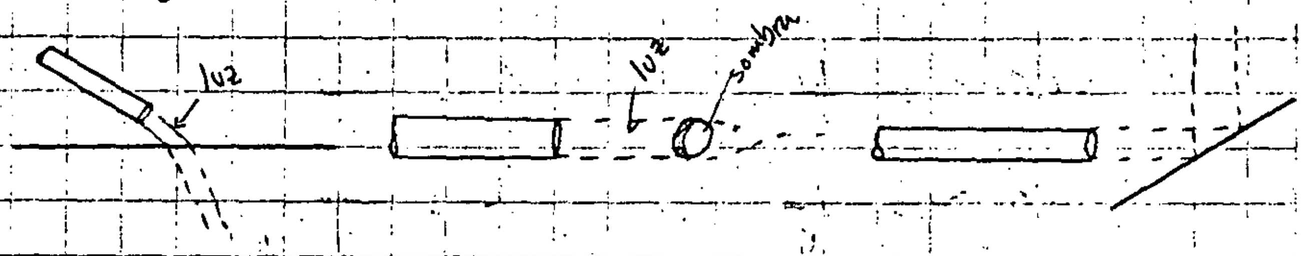
a) Refracción — c) Reflexión
b) Difracción — d) Rompiente.
e) Resistencia del fondo.



$d_1 > d_2$ $\quad C_1 > C_2$

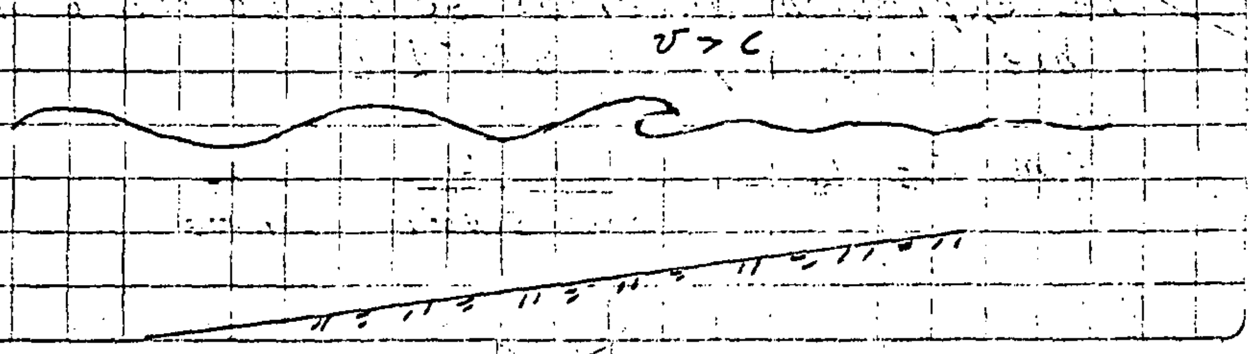
$L_1 > L_2$ $\quad T_1 = T_2 = T$

* Analogías con fenómenos de Luz



refracción — difracción — reflexión

rompiente



2.1 AMORTIGUAMIENTO DEL OLEAJE POR LA RESISTENCIA DEL FONDO

La energía del oleaje en aguas poco profundas disminuye debido a la fricción del fondo, ya que parte de ella se transforma en calor; esto tiene como consecuencia que se amortigue el oleaje.

Sea τ_b el esfuerzo cortante en el fondo, u_b la componente horizontal de la velocidad fuera de una delgada capa adyacente al fondo, entonces la potencia promedio disipada por unidad de área es

$$D = \overline{\tau_b u_b} \qquad 1$$

Suponiendo una capa límite turbulenta

$$\tau_b = C_r \rho u_b |u_b| \qquad 2$$

en la cual, C_r es el coeficiente de resistencia (adimensional) que es función de la razón vertical de desplazamiento de las partículas, de la rugosidad de fondo y del número de Reynolds de la capa límite, pero para fines prácticos puede considerarse

$$C_r = 0.01 \qquad 3$$

Si se sustituye la ec. A.3.26 y 2 en la 1 se obtiene

$$D = \frac{4}{3\pi} C_r \rho \left(\frac{\sigma a}{\operatorname{senh} kd}\right)^3 \qquad 4$$

Por otra parte, la razón de la energía transferida desde x_1 a $x_2 = x_1 + \Delta x$ es representada por P_1 y P_2, donde $P_2 = P_1 + \frac{dP}{dx}\Delta x$. La diferencia $P_1 - P_2$ es igual a la potencia disipada sobre la longitud Δx, la cual es igual a $D\Delta x$ (por unidad de ancho) así que

$$\frac{dP}{dx} + D = 0 \qquad 5$$

Sustituyendo 4, A.3.52 y A.3.49 se encuentra

$$\rho g n C a \frac{da}{dx} + \frac{4}{3\pi} C_r \rho \left(\frac{\sigma a}{\operatorname{senh} kd}\right)^3 = 0 \qquad 6$$

la cual puede ser escrita como

$$\frac{da}{a^2} + \beta dx = 0 \qquad 7$$

siendo

$$\beta = \frac{4}{3\pi} \frac{C_r}{gnC} \left(\frac{\sigma}{\operatorname{senh} kd}\right)^3 \qquad 8$$

o bien usando A.3.23

$$\beta = \frac{4}{3\pi} C_r \frac{k^2}{n \operatorname{senh}^2 kd \cosh kd} \qquad 9$$

Finalmente la integración de 7 da

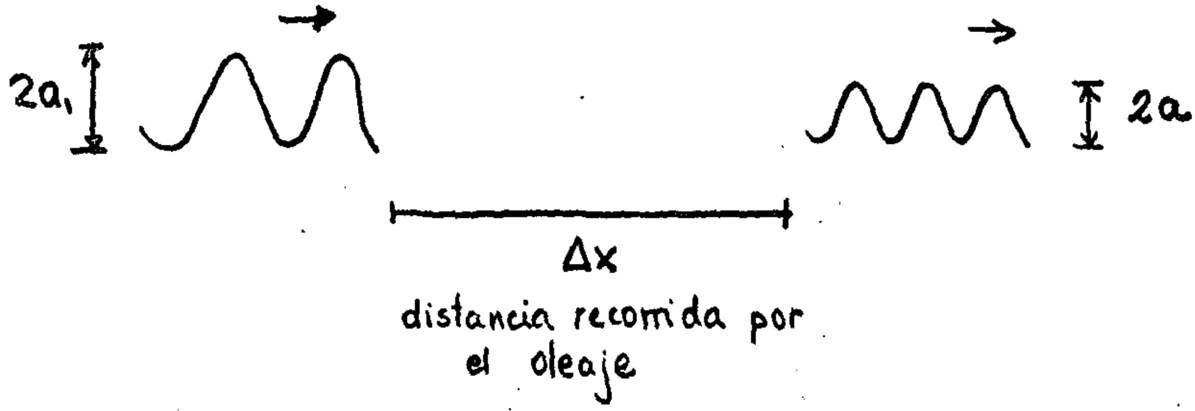
$$\frac{1}{a(x)} - \frac{1}{a(x_1)} = \beta (x - x_1)$$

que tambien es

$$\boxed{a = \frac{a_1}{1 + \beta a_1 \Delta x}} \qquad 10$$

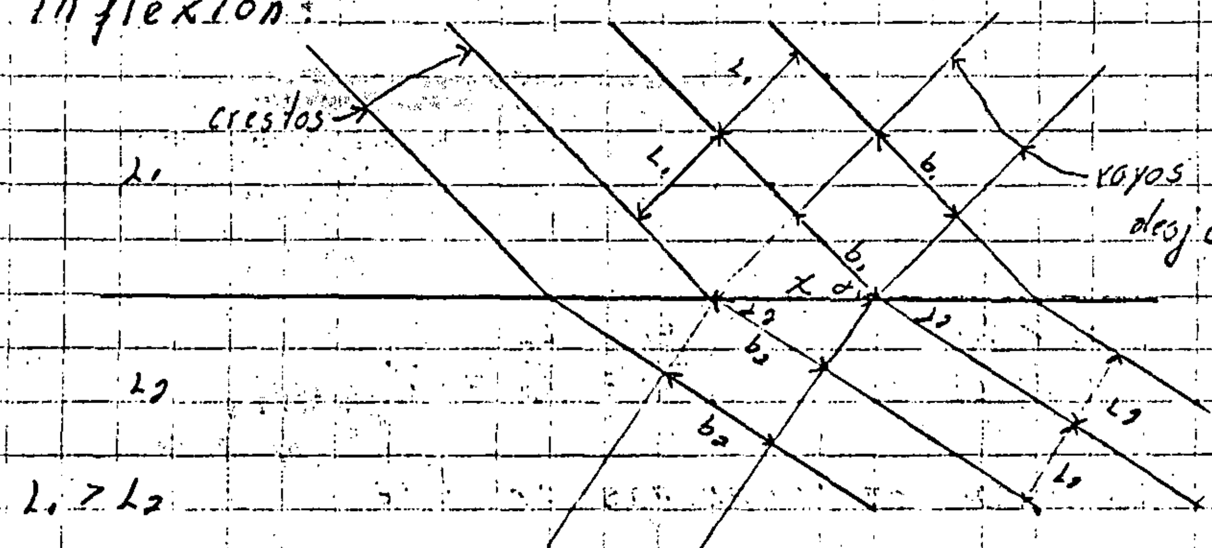
en la cual $a = a(x)$, $a_1 = a(x_1)$ y $\Delta x = x - x_1$.

Por inspección de la ec. 9 se tiene que a medida que disminuye la profundidad, β se aproxima a $\frac{4}{3\pi} \frac{C_r}{d^2}$



2.2 Refracción:

Cuando la luz penetra en una sustancia con un ángulo diferente a la normal de su superficie, la luz cambia de dirección produciendo lo que se llama refracción ó inflexión.



$$\operatorname{sen} \alpha_1 = \frac{L_1}{x} \qquad x = \frac{L_1}{\operatorname{sen} \alpha_1}$$

$$\operatorname{sen} \alpha_2 = \frac{L_2}{x} \qquad x = \frac{L_2}{\operatorname{sen} \alpha_2}$$

como $x = x$

$$\boxed{\frac{L_1}{\operatorname{sen} \alpha_1} = \frac{L_2}{\operatorname{sen} \alpha_2}}$$

La expresión anterior se conoce con el nombre de la ley de SNELL

com $c = \frac{L}{T}$ $\qquad \frac{c_1}{\operatorname{sen} \alpha_1} = \frac{c_2}{\operatorname{sen} \alpha_2}$

Potencia promedio en un periodo (por long.) unitario de cresta

$$\bar{P} = \frac{nE}{T} = \frac{n}{T}\frac{\gamma H^2 L}{8} = \frac{n\gamma H^2 C}{8}$$

La "potencia promedio en un periodo" entre ortogonales es constante

$$\bar{P}_1 = \bar{P}_2$$

$$\frac{\gamma n_1 H_1^2 C_1}{8} = \frac{\gamma n_2 H_2^2 C_2}{8}$$

En la práctica se parte de la condición de aguas profundas

$$\bar{P}_0 B_0 = \bar{P}_1 B_1 \qquad \frac{\gamma n_0 H_0^2 C_0}{8} = \frac{\gamma n_1 H_1^2 C_1}{8}$$

Como $\gamma = \gamma$ y $n_0 = 0.5$

$$H_1^2 = \frac{0.5 C_0}{n_1 C_1}\frac{B_0}{B_1} H_0^2$$

$$\boxed{H_1 = \sqrt{\frac{0.5 C_0}{n_1 C_1}}\sqrt{\frac{B_0}{B_1}}\, H_0}$$

$\sqrt{\frac{0.5 C_0}{n_1 C_1}}$ = coeficiente de fondo; D_d

$\sqrt{\frac{B_0}{B_1}}$ = coeficiente de refracción; K_d

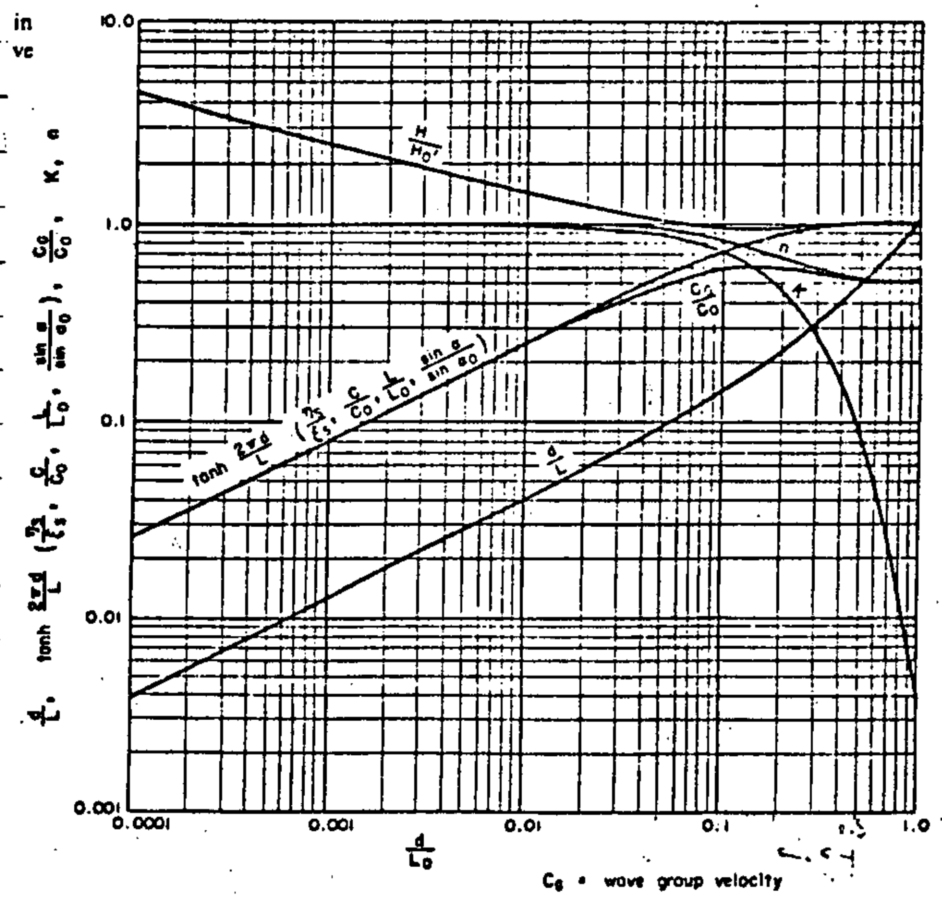
Cuando se da el caso en que $B_1 = B_2$ (no hay cambio en la dirección

$$H_2 = \sqrt{\frac{n_1 C_1}{n_2 C_2}}\, H_1 \qquad H_0' = H_1 = \sqrt{\frac{n_2 C_0}{n_1 C_1}}\, H_0$$

En general se tienen dos tipos de fondo

{ - curvas batimétricas irregulares
{ - curvas batimétricas rectas y paralelas.

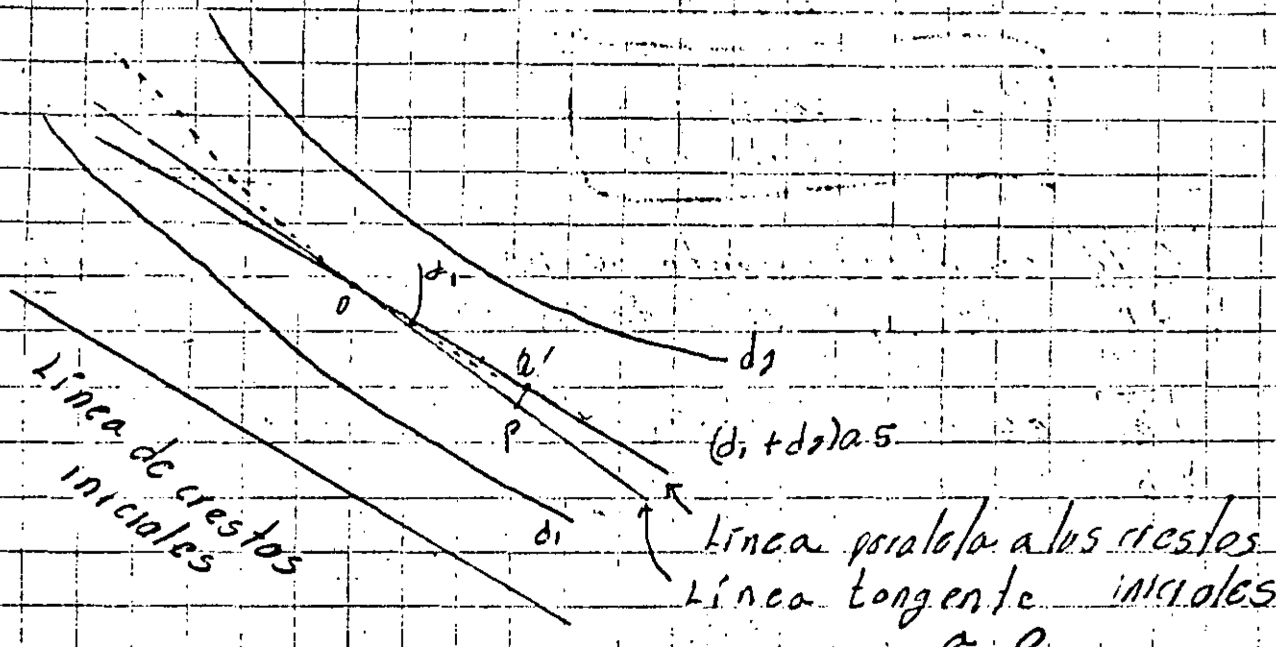
Análisis para el segundo caso



H = wave height
L = wave length
C = wave phase velocity
a = angle of wave crest with bottom
d = water depth
K = pressure response factor at bottom

C_G = wave group velocity
n = ratio of wave group velocity to phase velocity
-' = superscript refers to waves not affected by refraction
-$_0$ = subscript refers to deep water conditions
η_s, ξ_s = vertical and horizontal components surface water particle displacements

Método semigráfico



$\operatorname{sen}\alpha_1 = \frac{\overline{a'P}}{\overline{OP}}$ $\qquad$ $\operatorname{sen}\alpha_1 = \frac{C_1}{C_2}\operatorname{sen}\alpha_2$

con d_1 se obtiene C_1
con d_2 se obtiene C_2
de la fig. se obtiene α_1 y $\operatorname{sen}\alpha_2 = \frac{C_2}{C_1}\operatorname{sen}\alpha_1$

plano batimétrico

4/Enero/85

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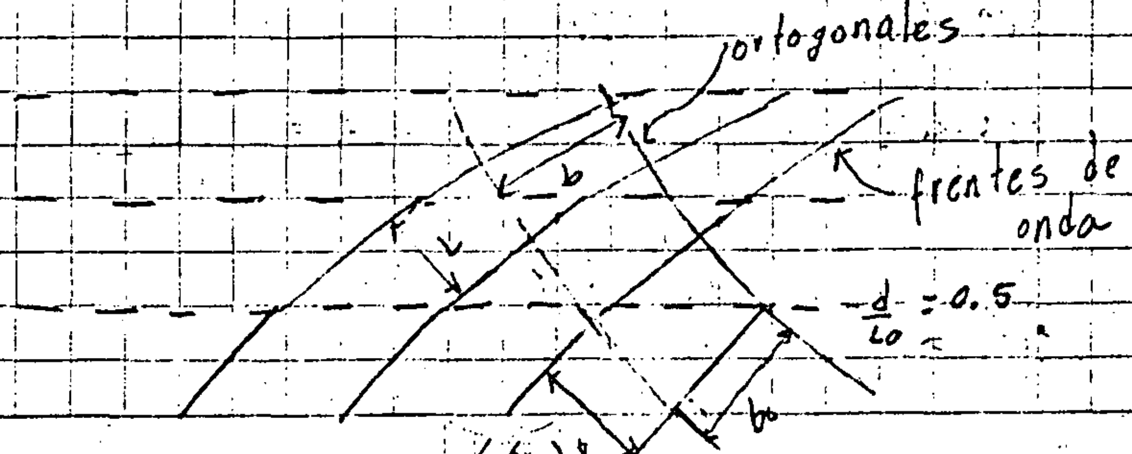
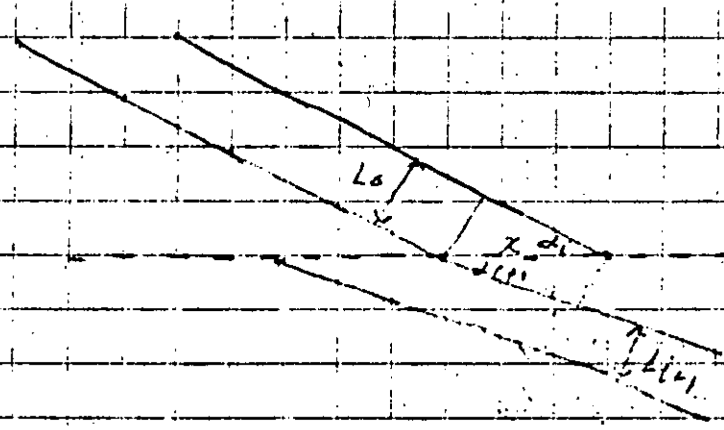


Las ondas se comienzan a refractar a partir de la profundidad $d = 0.5 L_o$

$$\boxed{H = H_o \sqrt{\frac{C_o/C}{2n}} \sqrt{\frac{b_o}{b}}}$$

$\sqrt{\frac{C_o/C}{2n}}$ es el coeficiente de fondo y se puede obtener del apéndice de Wiegel

$\sqrt{\frac{b_o}{b}}$ es el coeficiente de refracción



$$sen\,\alpha_i = \frac{L_i}{x} = \frac{C_i T}{x} \qquad sen\,\alpha_{i+1} = \frac{L_{i+1}}{x} = \frac{L_{i+1} T}{x}$$

$$\frac{sen\,\alpha_i}{sen\,\alpha_{i+1}} = \frac{L_i}{L_{i+1}} = \frac{C_i}{C_{i+1}}$$

$$\cos\alpha_i = \frac{b_i}{x} \qquad \cos\alpha_{i+1} = \frac{b_{i+1}}{x}$$

$$\frac{\cos\alpha_i}{\cos\alpha_{i+1}} = \frac{b_i}{b_{i+1}}$$

$$\frac{sen\,\alpha_0}{sen\,\alpha_1} = \frac{c_0}{c_1} \quad ; \quad sen\,\alpha_1 = \frac{c_1}{c_0}\, sen\,\alpha_0$$

$$\frac{sen\,\alpha_1}{sen\,\alpha_2} = \frac{c_1}{c_2} \quad ; \quad sen\,\alpha_2 = \frac{c_2}{c_1}\, sen\,\alpha_1$$

$$\frac{sen\,\alpha_2}{sen\,\alpha_3} = \frac{c_2}{c_3} \quad ; \quad sen\,\alpha_3 = \frac{c_3}{c_2}\, sen\,\alpha_2$$

$$\frac{sen\,\alpha_3}{sen\,\alpha_4} = \frac{c_3}{c_4} \quad ; \quad sen\,\alpha_4 = \frac{c_4}{c_3}\, sen\,\alpha_3$$

$$sen\,\alpha_4 = \frac{c_4}{c_3}\,\frac{c_3}{c_2}\,\frac{c_2}{c_1}\,\frac{c_1}{c_0}\, sen\,\alpha_0$$

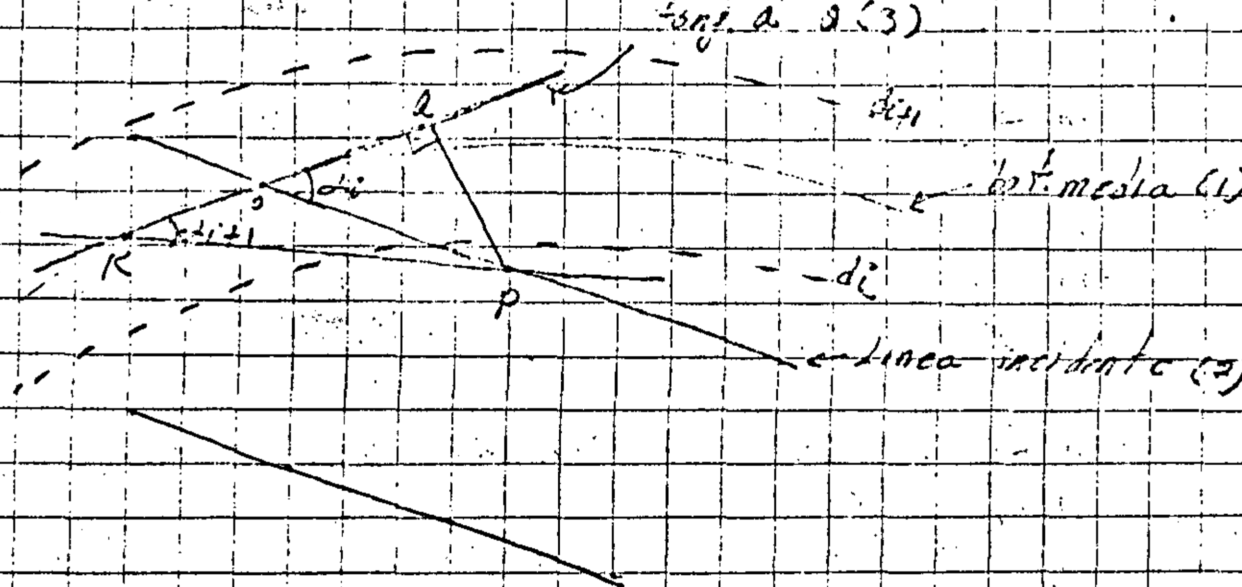
$$\therefore\ sen\,\alpha_4 = \frac{c_4}{c_0}\, sen\,\alpha_0$$

Esto es válido sólo para batimétricas rectas y paralelas

En general $\quad sen\,\alpha = \frac{c}{c_0}\, sen\,\alpha_0$

$$\boxed{\alpha = ang\ sen\left(\frac{c}{c_0}\, sen\,\alpha_0\right)}$$

$$\boxed{K_d = \sqrt{\frac{b_0}{b}} = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}}}$$ (coeficiente de ~~fondo~~ refracción)



$$\operatorname{sen} \alpha_i = \frac{QP}{OP} \qquad \alpha_{i+1} = \operatorname{ang\,sen} \frac{C_{i+1}}{C_i} \operatorname{sen} \alpha_i$$

$$\operatorname{sen} \alpha_{i+1} = \frac{QP}{RP} \; ; \quad \frac{\operatorname{sen} \alpha_i}{\operatorname{sen} \alpha_{i+1}} = \frac{RP}{OP} = \frac{C_i}{C_{i+1}}$$

$$RP = \frac{C_i}{C_{i+1}} \quad \text{si } OP \text{ es unitaria.}$$

Procedimiento

i	d_i	d_i/L_0	d_i/L	L_i	C_i	C_i/C_{i+1}
0	30	0.200	0.2251	133.27	13.60	1.125
1	20	0.133	0.1691	118.27	12.07	1.127
2	10	0.067	0.111	90	9.18	1.314

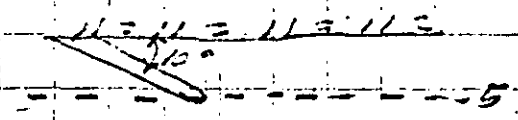
$L_0 = 150$ m $\quad C_0 = 15.3$ m/seg $\quad T = 9.8$ seg

Procedimiento:

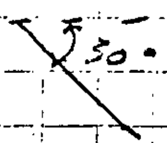
1. se fija un punto O
2. se traza una tangente por O
3. se obtiene el frente el punto P
4. se obtiene el punto R

Ejemplo 1:

En la batimétrica 5 m existe una estructura que conforma con la línea de costa un ángulo de 10°. Intereza conocer la dirección y la altura de la ola sobre ella sí en aguas profundas el oleaje tiene una longitud de de 100 m, $H_0 = 5$ m; y tiene una dirección de 30°. El fondo tiene una pendiente perpendicular a la línea de costa.



$$\frac{sen\, \alpha}{sen\, \alpha_0} = \frac{C}{C_0}$$



$\frac{d}{L_0} = 0.5$

$$sen\, \alpha = sen\, \alpha_0 \frac{C}{C_0}$$

$$\alpha = ang\, sen\left(sen\, \alpha_0 \frac{C}{C_0}\right)$$

$$K_d = \sqrt{\frac{\cos \alpha_0}{\cos \alpha}} \; ; \quad C_0 = \sqrt{\frac{g L_0}{2\pi}} = \sqrt{\frac{9.81(100)}{2\pi}} = 12.5\ m/seg$$

$$T = \frac{L_0}{C_0} = \frac{100}{12.5} = 8.1$$

$$C(5m) = ? \quad \frac{d}{L_0} = \frac{5}{100} = 0.05 \rightarrow \frac{d}{L} = 0.09416$$

$L = 53.10\ m \quad ; \quad C = \frac{L}{T} = \frac{53.10}{8} = 6.64\ m/sg$

$\alpha = \text{ang sen}\left(\frac{6.64}{12.5}\ \text{sen}\ 30\right) = 15.18$

$K_d = \sqrt{\frac{\cos 30}{\cos 15.18}} = 0.9472$

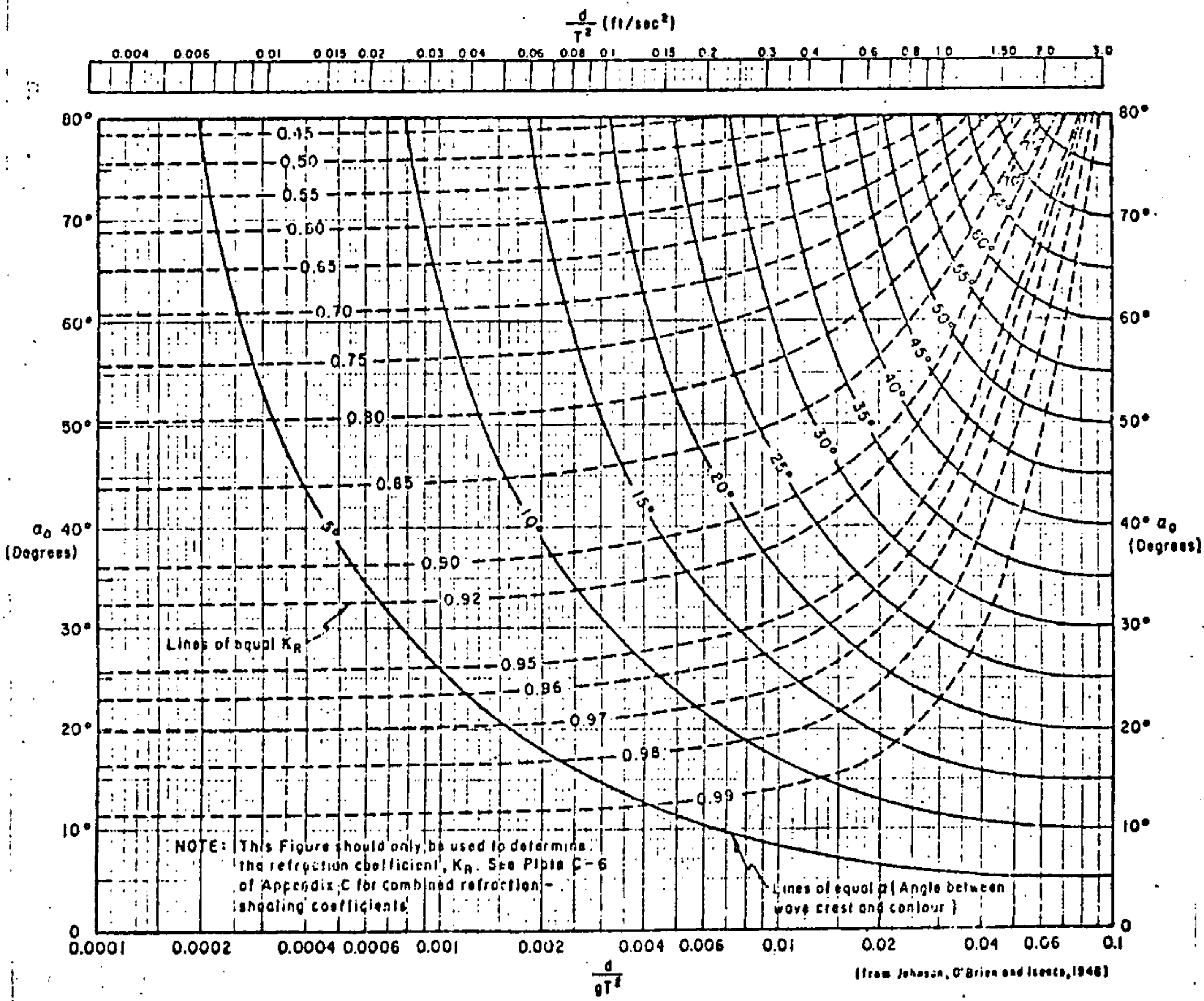


Figure 2-19. Changes in Wave Direction and Height Due to Refraction on Slopes with Straight, Parallel Depth Contours

Ejemplo 2.

Cálculo de la altura de ola, H para los puntos A, B y C. de la fig. 1

Datos: $T_o = 7$ seg , $H_o = 6m$

$L_o = \frac{gT^2}{2\pi} = \frac{9.81(7)^2}{2\pi} = 76.5 m$

$\frac{d_o}{L_o} = 0.5$; $d_o = 0.5 L_o = 0.5(76.5) = 38.25$ m

$C_o = \frac{L_o}{T_o} = \frac{76.5}{7} = 10.93$ m/seg.

i	d_i	d_i/L_o	d_i/L_i	L_i	C_i	C_i/C_o
1	35	0.4575	0.4604	76.02	10.86	1.006
2	30	0.3921	0.3975	75.47	10.78	1.007
3	25	0.3268	0.3364	74.32	10.62	1.015
4	20	0.2614	0.2780	71.94	10.28	1.033
5	15	0.1961	0.2219	67.60	9.66	1.064
6	10	0.1307	0.1672	59.81	8.54	1.131
7	5	0.0654	0.1096	45.62	6.52	1.310

Calculo del coeficiente de fondo D_d

$d = 2.5 m$; $\frac{d}{L_o} = \frac{2.5}{76.5} = 0.033 \Rightarrow \frac{d}{L} = 0.07507$

$L = 33.30 m$ y $D_d = 1.104$ (tablas de Wiegel)

- Calculo de H en A

$b_o = 1 m$; $b = 4.8 m$; $K_d = \sqrt{b_o/b} = \sqrt{1/4.8} = 0.456$

$H_A = D_d K_d H = (1.104)(0.456)(6)$

$\therefore H_A = 3.02$ m

.- Calculo de H en B

$b_0 = 4.7\ cm$, $b = 1.7\ cm$; $K_d = \sqrt{b_0/b} = \sqrt{4.7/1.7} = 1.663$

$H_B = H\,D_d\,K_d = 6\,(1.104)(1.663)$

$\therefore\ H_B \doteq 11.01\ m$

.- Calculo de H en C

$b_0 = 1\ cm$; $b = 0.9\ m$; $K_d = \sqrt{b_0/b} = \sqrt{1/0.9} = 1.054$

$H_C = H\,D_d\,K_d = 6\,(1.104)(1.054)$

$\therefore\ H_C = 6.98\ m$

.- Procedimiento alternativa para calcular Hc

de la figura $\alpha_0 = 12°$ $D_d = 1.104$

$C_0 = 10.93\ m/seg$; $L_{2.5} = 33.30$; $C_{2.5} = \frac{33.3}{7} = 4.757\ m/seg$

$\alpha = \text{ang sen}\left(\text{sen}\,\alpha_0\ \frac{C}{C_0}\right) = \text{ang sen}\left(\text{sen}\,12 \cdot \frac{4.757}{10.93}\right) = 5.192°$

$K_d = \sqrt{\frac{\cos \alpha_0}{\cos \alpha}} = \sqrt{\frac{\cos 12°}{\cos 5.192}} = 0.991$

$H_C = H\,D_d\,K_d = 6\,(1.104)(0.991)$

$H_C \doteq 6.57\ m$

$\frac{d}{g T^2} = \frac{2.5}{9.81(7)^2}$

ENCONTRAR LA DIRECCION DEL OLEAJE Y SU ALTURA EN LOS PUNTOS A, B Y C. Si en aguas profundas el oleaje tiene un periodo de 7 s, altura de 6 m y la dirección mostrada en esta figura.
(Para el punto C se puede considerar batimétricas rectas y paralelas)

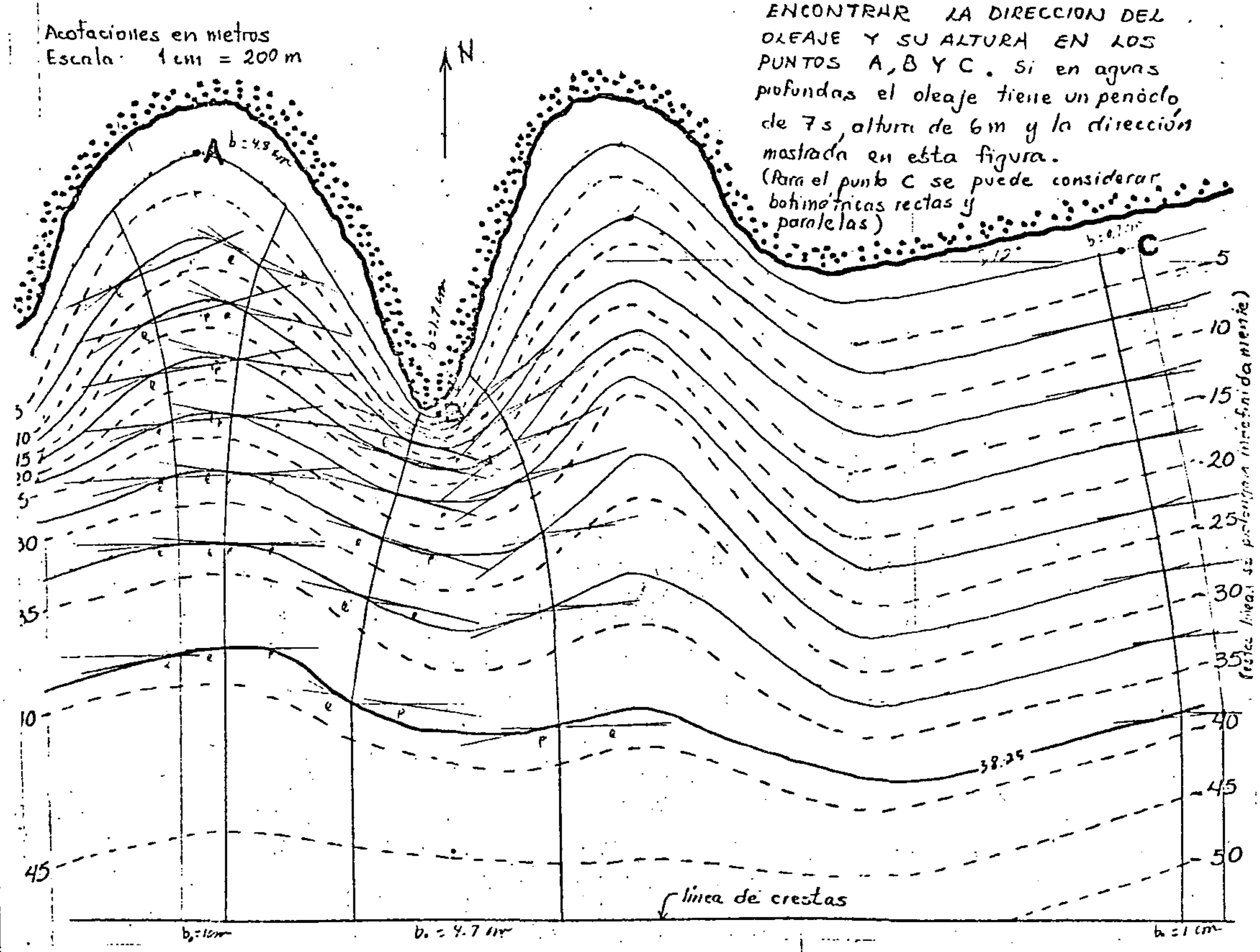


Fig 1

2.3 Difracción del oleaje

Hipótesis de partida

1- barreras semiinfinitas
2.- oleaje lineal
3.- profundidad constante

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (1)$$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=-d} = 0 \quad (2) \qquad \eta = -\frac{1}{g} \left.\frac{\partial \phi}{\partial t}\right|_{z=0} \qquad (3)$$

$$\phi = a\frac{\sigma}{k} \frac{\cosh k(z+d)}{\operatorname{senh} kd} \operatorname{sen}(kx - \sigma t)$$

se propone $\phi = A\, f(x,y) \cosh k(z+d) e^{i\sigma t}$

$$e^{i\sigma t} = \cos \sigma t + i \operatorname{sen} \sigma t$$

$f(x,y)$ es una función compleja

A es una constante de proporc. de la amplitud

$$\frac{\partial \phi}{\partial x} = A \cosh k(z+d) e^{i\sigma t} \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial x^2} = A \cosh k(z+d) e^{i\sigma t} \frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 \phi}{\partial y^2} = A \cosh k(z+d) e^{i\sigma t} \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial \phi}{\partial z} = A\, f(x,y)\, e^{i\sigma t}\, k \operatorname{senh} k(z+d)$$

$$\frac{\partial^2 \phi}{\partial z^2} = A\, f(x,y)\, e^{i\sigma t}\, k^2 \cosh k(z+d)$$

sust. en 1

$$A \cosh k(z+d)\, e^{i\sigma t} \frac{\partial^2 f}{\partial x^2} + A \cosh k(z+d)\, e^{i\sigma t} \frac{\partial^2 f}{\partial y^2} +$$

$$+ A \cosh k(z+d)\, e^{i\sigma t} k^2 f = 0$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0}$$ Ec. de Helmholtz (4)

de 3 $\quad \eta = -\frac{1}{g} A f \cosh k(z+d)\, i\sigma e^{i\sigma t} \Big|_{z=0}$

$$\boxed{\eta = -\frac{A\sigma f i}{g} \cosh kd\, e^{i\sigma t}}$$

Oleaje incidente (propagación en la dirección y)

$$f = e^{iky} \qquad (6)$$

$$\eta_i = -\frac{A\sigma i}{g} \cosh kd\, e^{i(ky+\sigma t)} \qquad (7)$$

al dividir (7) entre (5)

$$\frac{\eta}{\eta_i} = f e^{iky} \qquad \frac{H}{H_i} = |f| = K_{dif}$$

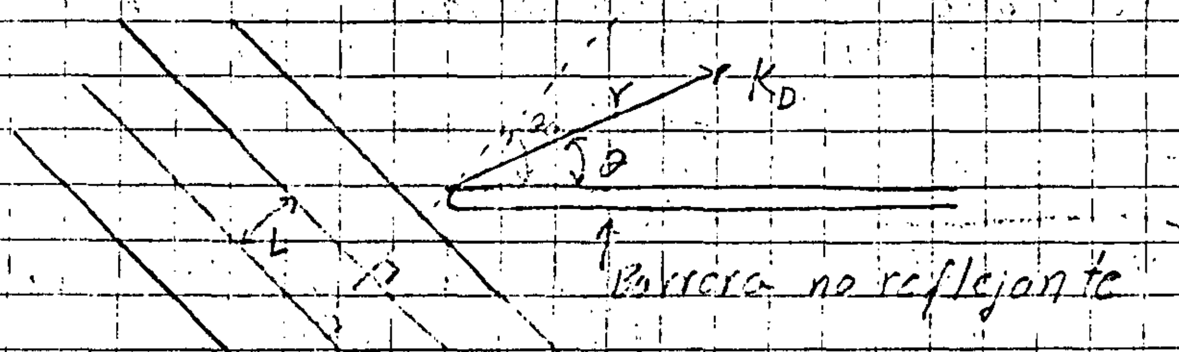
Table 8.1. DIFFRACTION COEFFICIENTS, K', AS A FUNCTION OF INCIDENT WAVE ANGLE, θ_0, AND POSITION, r/L AND θ, (from Wiegel, 1962)

Value of r/L	Value of θ (degrees)												
	0	15	30	45	60	75	90	105	120	135	150	165	180
							$\theta_0 = 15°$						
1/2	0.49	0.72	0.83	0.90	0.97	1.01	1.03	1.02	1.01	0.99	0.99	1.00	1.00
1	0.38	0.73	0.83	0.95	1.04	1.04	0.99	0.98	1.01	1.01	1.00	1.00	1.00
2	0.21	0.68	0.86	1.05	1.03	0.97	1.02	0.99	1.00	1.00	1.00	1.00	1.00
5	0.13	0.63	0.99	1.04	1.03	1.02	0.99	0.99	1.00	1.01	1.00	1.00	1.00
10	0.35	0.58	1.10	1.05	0.98	0.99	1.01	1.00	1.00	1.00	1.00	1.00	1.00
							$\theta_0 = 30°$						
1/2	0.61	0.63	0.68	0.76	0.87	0.97	1.03	1.05	1.03	1.01	0.99	0.95	1.00
1	0.50	0.53	0.63	0.78	0.95	1.06	1.05	0.98	0.98	1.01	1.01	0.97	1.00
2	0.40	0.44	0.59	0.84	1.07	1.03	0.96	1.02	0.98	1.01	0.99	0.95	1.00
5	0.27	0.32	0.55	1.00	1.04	1.04	1.02	0.99	0.99	1.00	1.01	0.97	1.00
10	0.20	0.24	0.54	1.12	1.06	0.97	0.99	1.01	1.00	1.00	1.00	0.98	1.00
							$\theta_0 = 45°$						
1/2	0.49	0.50	0.55	0.63	0.73	0.85	0.96	1.04	1.06	1.04	1.00	0.99	1.00
1	0.38	0.40	0.47	0.59	0.76	0.95	1.07	1.06	0.98	0.97	1.01	1.01	1.00
2	0.29	0.31	0.39	0.56	0.83	1.08	1.04	0.96	1.03	0.98	1.01	1.00	1.00
5	0.18	0.20	0.29	0.54	1.01	1.04	1.05	1.03	1.00	0.99	1.01	1.00	1.00
10	0.13	0.15	0.22	0.53	1.13	1.07	0.96	0.98	1.02	0.99	1.00	1.00	1.00
							$\theta_0 = 60°$						
1/2	0.40	0.41	0.45	0.52	0.60	0.72	0.85	1.13	1.04	1.06	1.03	1.01	1.00
1	0.31	0.32	0.36	0.44	0.57	0.75	0.96	1.08	1.06	0.98	0.98	1.01	1.00
2	0.22	0.23	0.28	0.37	0.55	0.83	1.08	1.04	0.96	1.03	0.98	1.01	1.00
5	0.14	0.15	0.18	0.28	0.53	1.01	1.04	1.05	1.03	0.99	0.99	1.00	1.00
10	0.10	0.11	0.13	0.21	0.52	1.14	1.07	0.96	0.98	1.01	1.00	1.00	1.00
							$\theta_0 = 75°$						
1/2	0.34	0.35	0.38	0.42	0.50	0.59	0.71	0.85	0.97	1.04	1.05	1.02	1.00
1	0.25	0.26	0.29	0.34	0.43	0.56	0.75	0.95	1.02	1.06	0.98	0.98	1.00
2	0.18	0.19	0.22	0.26	0.36	0.54	0.83	1.09	1.04	0.96	1.03	0.99	1.00
5	0.12	0.12	0.13	0.17	0.27	0.52	1.01	1.04	1.05	1.03	0.99	0.99	1.00
10	0.08	0.08	0.10	0.13	0.20	0.52	1.14	1.07	0.96	0.98	1.01	1.00	1.00
							$\theta_0 = 90°$						
1/2	0.31	0.31	0.33	0.36	0.41	0.49	0.59	0.71	0.85	0.96	1.03	1.03	1.00
1	0.22	0.23	0.24	0.28	0.33	0.42	0.56	0.75	0.96	1.07	1.05	0.99	1.00
2	0.16	0.16	0.18	0.20	0.25	0.35	0.54	0.69	1.08	1.04	0.96	1.02	1.00
5	0.10	0.10	0.11	0.13	0.16	0.27	0.53	1.01	1.04	1.05	1.02	0.99	1.00
10	0.07	0.07	0.08	0.09	0.13	0.20	0.52	1.14	1.07	0.96	0.99	1.01	1.00
							$\theta_0 = 105°$						
1/2	0.28	0.28	0.29	0.32	0.35	0.41	0.49	0.59	0.72	0.85	0.97	1.01	1.00
1	0.20	0.20	0.24	0.23	0.27	0.33	0.42	0.56	0.75	0.95	1.06	1.04	1.00
2	0.14	0.14	0.13	0.17	0.20	0.25	0.35	0.54	0.83	1.08	1.03	0.97	1.00
5	0.09	0.09	0.10	0.11	0.13	0.17	0.27	0.52	1.02	1.04	1.04	1.02	1.00
10	0.07	0.06	0.08	0.08	0.09	0.12	0.20	0.52	1.14	1.07	0.97	0.99	1.00
							$\theta_0 = 120°$						
1/2	0.25	0.26	0.27	0.28	0.31	0.35	0.41	0.50	0.60	0.73	0.87	0.97	1.00
1	0.18	0.19	0.19	0.21	0.23	0.27	0.33	0.43	0.57	0.76	0.95	1.04	1.00
2	0.13	0.13	0.14	0.14	0.17	0.20	0.26	0.36	0.55	0.83	1.07	1.03	1.00
5	0.08	0.08	0.08	0.09	0.11	0.13	0.16	0.27	0.53	1.01	1.04	1.03	1.00
10	0.06	0.06	0.06	0.07	0.07	0.09	0.13	0.20	0.52	1.13	1.06	0.98	1.00
							$\theta_0 = 135°$						
1/2	0.24	0.24	0.25	0.26	0.28	0.32	0.36	0.42	0.52	0.63	0.76	0.90	1.00
1	0.18	0.17	0.18	0.19	0.21	0.23	0.28	0.34	0.44	0.59	0.78	0.95	1.00
2	0.12	0.12	0.13	0.14	0.14	0.17	0.20	0.26	0.37	0.56	0.84	1.05	1.00
5	0.08	0.07	0.08	0.08	0.09	0.11	0.13	0.17	0.28	0.54	1.00	1.04	1.00
10	0.05	0.06	0.06	0.06	0.07	0.08	0.09	0.13	0.21	0.53	1.12	1.05	1.00
							$\theta_0 = 150°$						
1/2	0.23	0.23	0.24	0.25	0.27	0.29	0.33	0.38	0.45	0.55	0.68	0.83	1.00
1	0.16	0.17	0.17	0.18	0.19	0.22	0.24	0.29	0.36	0.47	0.63	0.83	1.00
2	0.12	0.12	0.12	0.13	0.14	0.15	0.18	0.22	0.28	0.39	0.59	0.86	1.00
5	0.07	0.07	0.08	0.08	0.08	0.10	0.11	0.13	0.18	0.29	0.55	0.99	1.00
10	0.05	0.05	0.05	0.06	0.06	0.07	0.08	0.10	0.13	0.22	0.54	1.10	1.00

Table 8.1. (cont.)

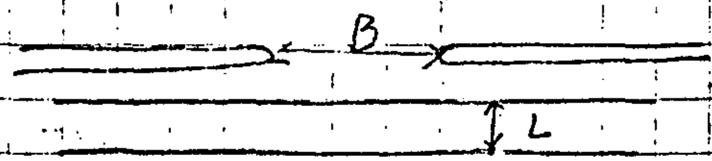
						$\theta_0 = 165°$							
1/2	0.23	0.23	0.23	0.24	0.26	0.28	0.31	0.35	0.41	0.50	0.63	0.79	1.00
1	0.16	0.16	0.17	0.17	0.19	0.20	0.23	0.26	0.32	0.40	0.53	0.73	1.00
2	0.11	0.11	0.12	0.12	0.13	0.14	0.16	0.19	0.23	0.31	0.44	0.68	1.00
5	0.07	0.07	0.07	0.07	0.08	0.09	0.10	0.12	0.15	0.20	0.32	0.63	1.00
10	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.08	0.11	0.11	0.21	0.58	1.00
						$\theta_0 = 180°$							
1/2	0.20	0.25	0.23	0.24	0.25	0.28	0.31	0.34	0.40	0.49	0.61	0.78	1.00
1	0.10	0.17	0.16	0.18	0.18	0.23	0.22	0.25	0.31	0.38	0.50	0.70	1.00
2	0.02	0.09	0.12	0.12	0.13	0.18	0.16	0.18	0.22	0.29	0.40	0.60	1.00
5	0.02	0.06	0.07	0.07	0.07	0.08	0.10	0.12	0.14	0.18	0.27	0.46	1.00
10	0.01	0.05	0.05	0.04	0.06	0.07	0.07	0.08	0.10	0.13	0.20	0.36	1.00

Difracción para una barrera semi-infinita.

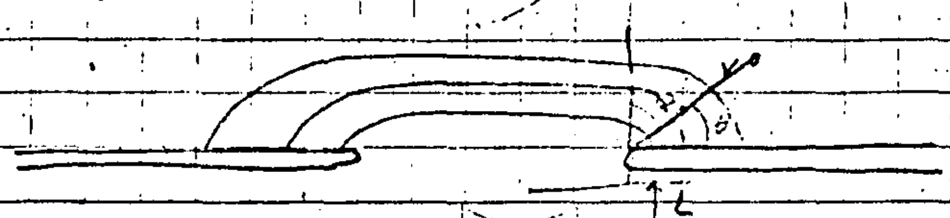


Se entra a los renglones correspondientes de (la tabla 8.1) θ_o, Luego con la relación r/L y θ se encuentra K_D

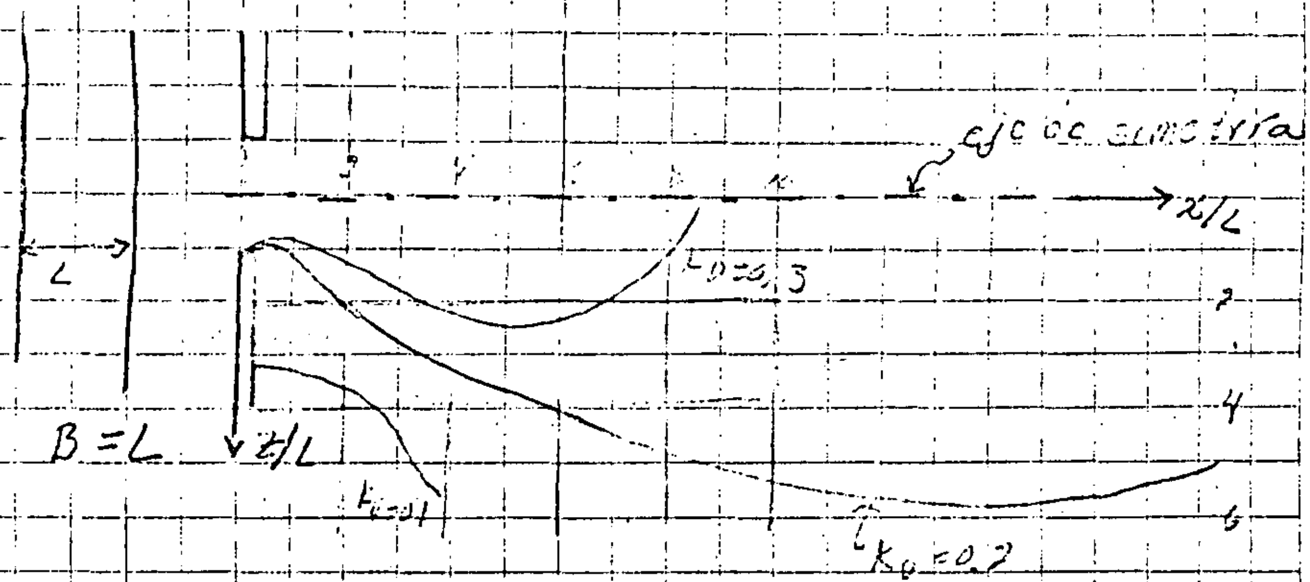
Difracción para una abertura en una barrera



1° Caso si $B > 5L$ se analiza por tramos separados

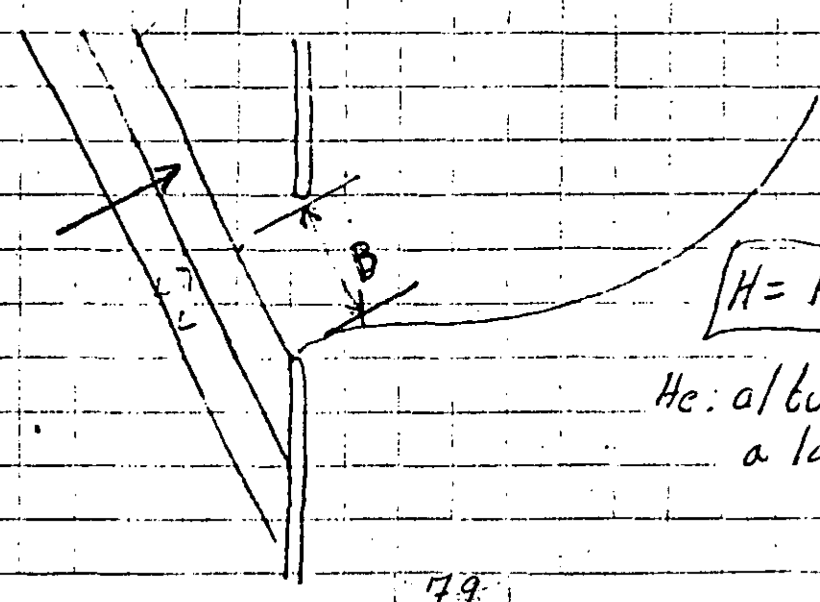


B) si $B \leq 5L$



En todos los casos anteriores, los frentes de onda son paralelos a el ancho

Para el caso en que el frente de onda no sea paralelo al ancho se hace la siguiente consideración.



$\boxed{H = K_0 H_e}$

He: altura de la ola a la entrada.

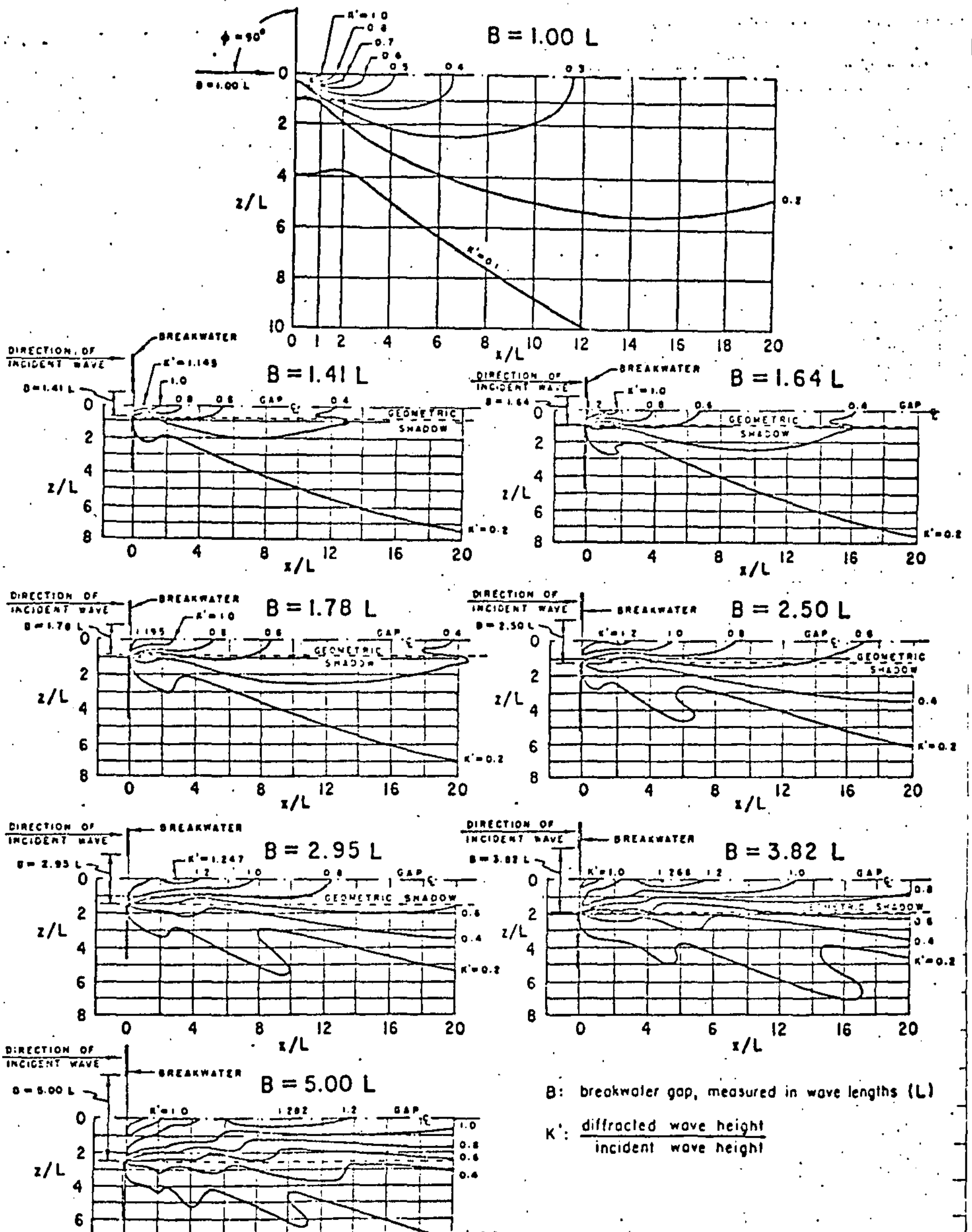
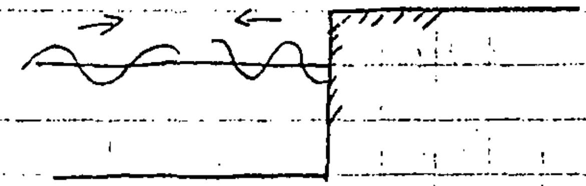


Fig. 8.12. Diffraction of waves at breakwater gap—contours of equal diffraction coefficient (after Johnson, 1952)

2.4 Reflexión.

Cuando una onda choca con un obstáculo se puede presentar otra onda que parta desde este; este fenómeno corresponde a la reflexión.

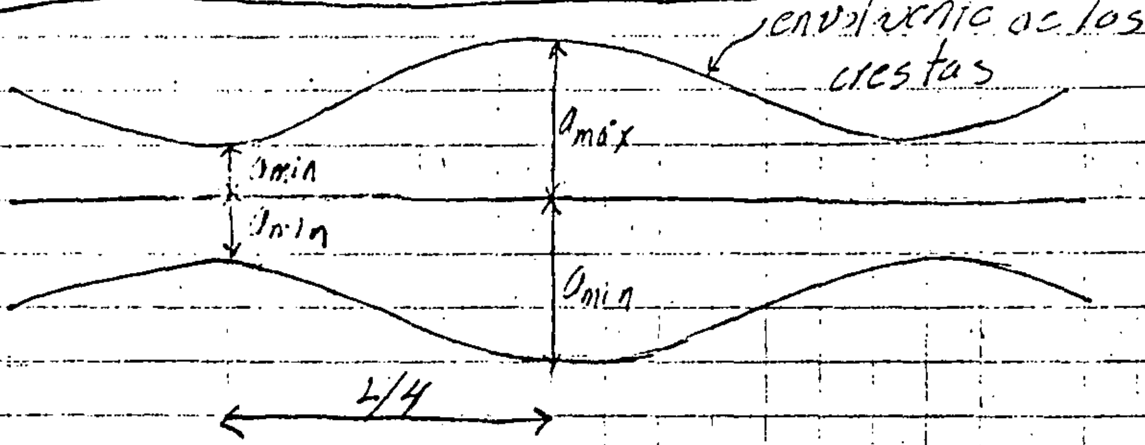
$\eta_r = a_r \cos(kx + \sigma t)$. $\eta_i = a_i \cos(kx + \sigma t)$



$\eta = a_i \cos(kx + \sigma t) + a_r \cos(kx + \sigma t)$

POR UNA IDENTIDAD DE LAS FUNCIONES CIRCULARES:

$$\boxed{\eta = (a_i + a_r)\cos kx \cos \sigma t + (a_i - a_r)\,\mathrm{sen}\, kx\; \mathrm{sen}\, \sigma t}$$



$a_{máx} = a_i + a_r$; $a_{min} = a_i - a_r$

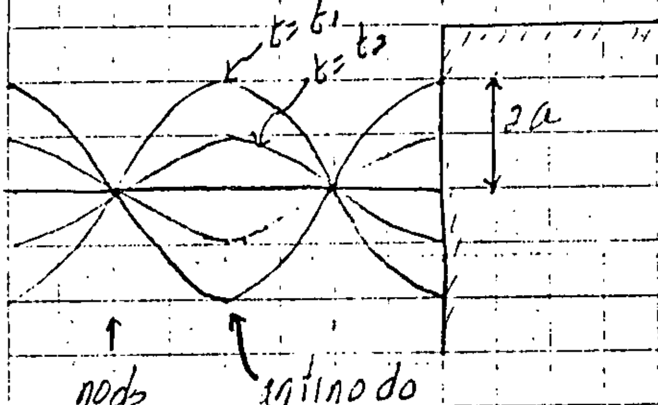
de aquí $a_i = \dfrac{a_{max} + a_{min}}{2}$ $\quad a_r = \dfrac{a_{máx} - a_{min}}{2}$

Coeficiente de reflexión $K_r = \dfrac{a_r}{a_i} = \dfrac{a_{máx} - a_{min}}{a_{max} + a_{min}}$

Lo anterior corresponde a una reflexión parcial y se tiene $a_{min} \neq 0$

La reflexión total ocurre cuando $a_i = a_r = a$

$$\boxed{\eta = 2a \cos kx \cos \sigma t}$$



La función potencial ϕ es

$$\phi = \phi_i + \phi_r \qquad a_i = a_r$$

$$\phi = \frac{a_i \sigma}{k} \frac{\cosh k(z+d)}{\operatorname{senh} kd} \operatorname{sen}(kx - \sigma t)$$

$$+ \frac{a \sigma}{k} \frac{\cosh k(z+d)}{\operatorname{senh} kd} \operatorname{sen}(kx + \sigma t)$$

ó sea $\quad \phi = 2a \dfrac{\sigma}{k} \dfrac{\cosh k(z+d)}{\operatorname{senh} kd} \operatorname{sen} kx \cos \sigma t$

$$u = \frac{\partial \phi}{\partial x}$$

coeficientes de reflexión

tipos de estructura	K_r
Muros verticales (casi verticales)	0.8 - 1
Estructuras taludes 2 y 3	0.4 - 0.8
Estructuras de bloques de concreto	0.3 - 0.4
Terraplén	0.3 - 0.5
playas	0.1 - 0.2

Reflexión del oleaje en playas.

$K_L = \chi_1 \chi_2$

χ_1 depende de la rugosidad y permeabilidad de la playa

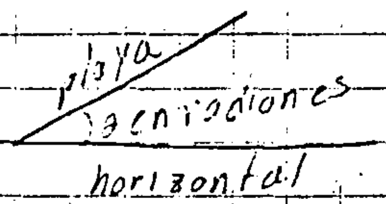
0.8 para playas lisas impermeables

0.3 - 0.6 para playas rugosas impermeables

χ_2 depende de la pendiente de la playa y esbeltez (H_0/L_0) del oleaje en aguas profundas.

Para valuar χ_2 obtengase primero

$$\left(\frac{H_0}{L_0}\right)_{max} = \left(\frac{2\theta}{\pi}\right)^{1/2} \frac{sen^2\theta}{\pi}$$

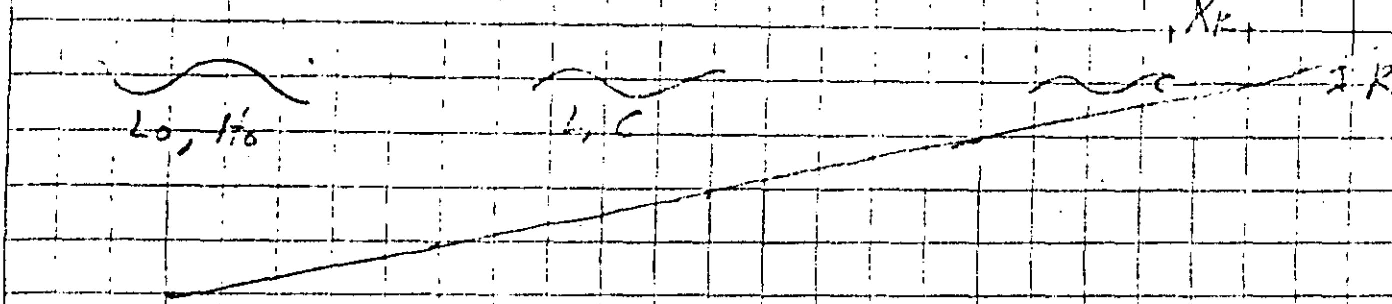


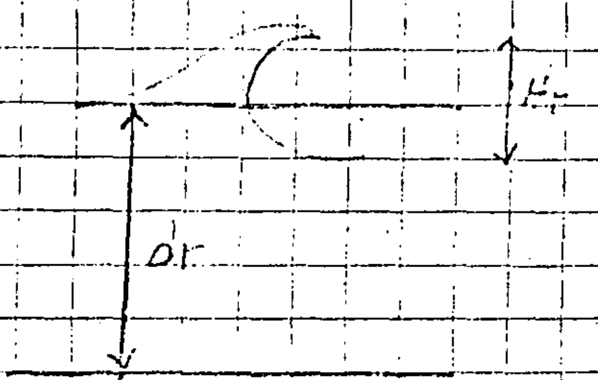
$$\chi_2 = \frac{(H_0/L_0)_{max}}{(H_0/L_0)}$$

sólo si se tiene un valor menor de 1

en caso contrario dese a χ_2 el valor de 1.

2.5 Rompiente del oleaje





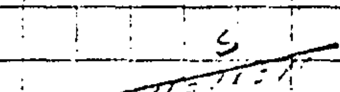
★ La condición para que la ola rompa es

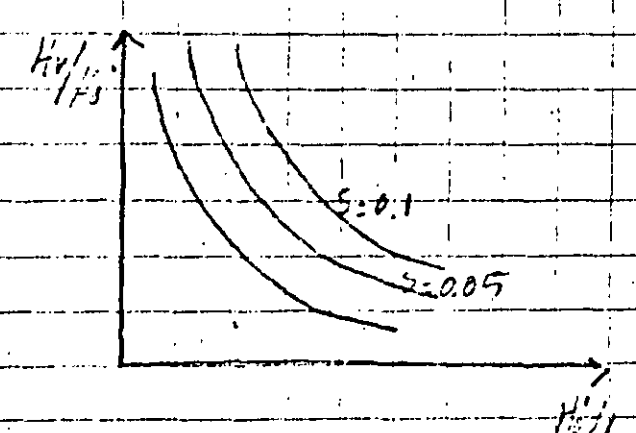
$$u > C$$

u: vel. horizontal de las partículas.

La forma en que rompe la ola se clasifica en los siguientes tipos:
continua, rodante y ondulante

Criterio de Goda

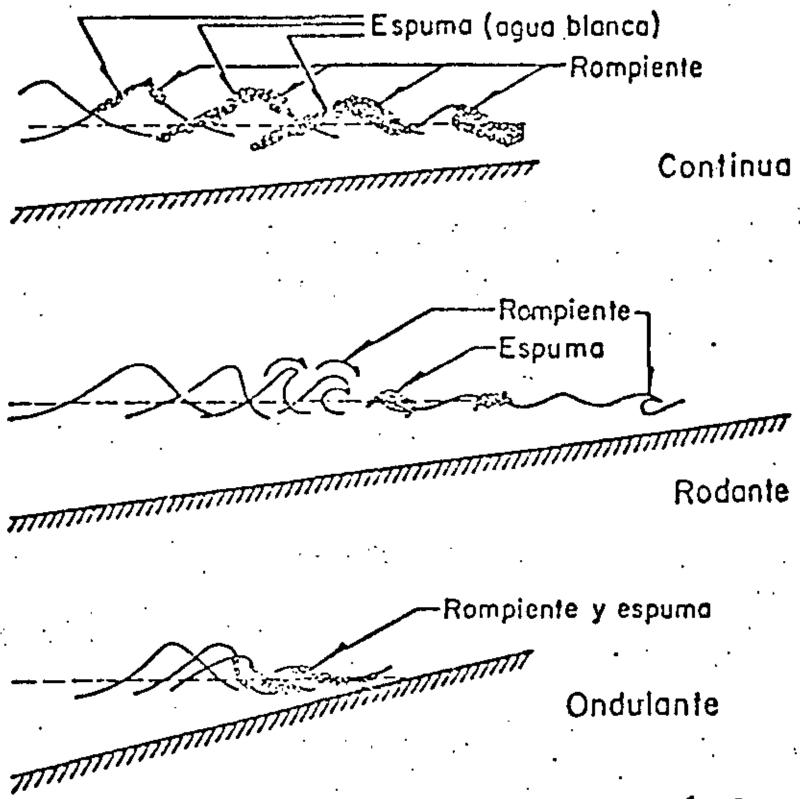




Como $H_{ref} = H_0 D_d K_d$

$D_d = H/H_0' \therefore H = H_0 \frac{H}{H_0'} K_d$

$\therefore \boxed{H_0' = H_0 K_d}$



Clasificación del oleaje rompiente, según Sorensen

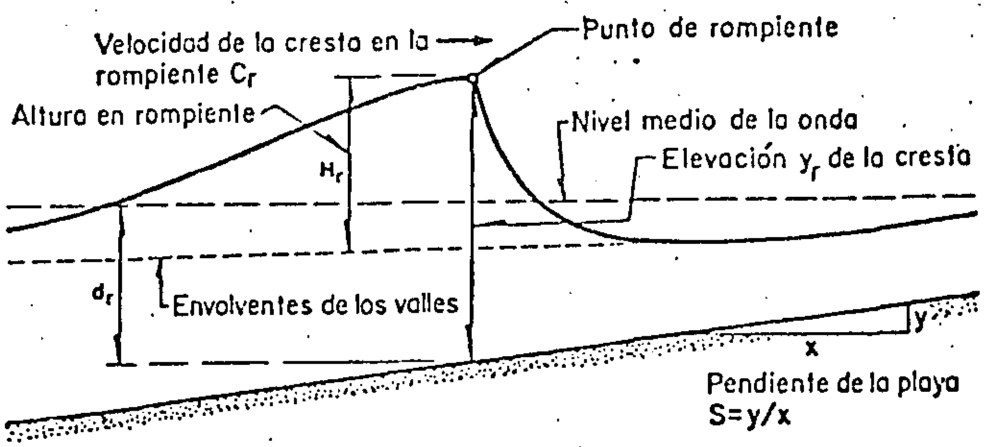


FIG. Términos utilizados para definir a la ola rompiente, según Iversen (1952)

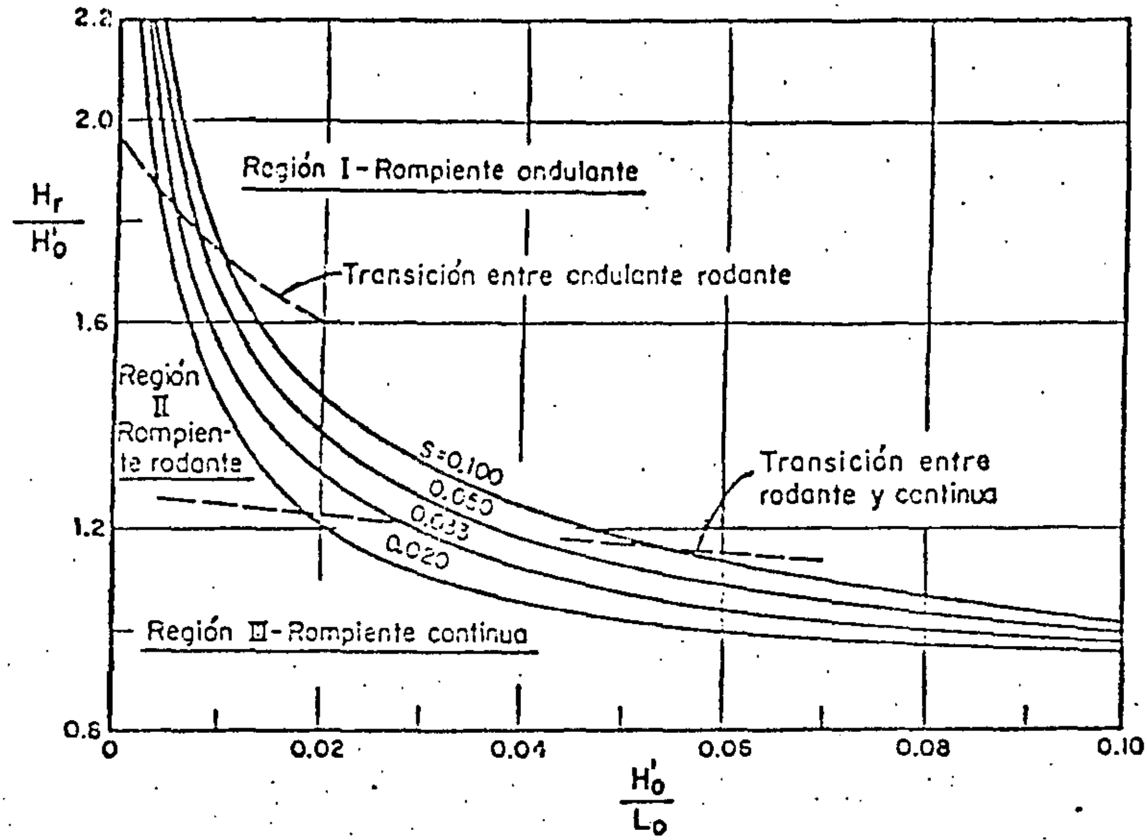


FIG. II.3 Altura de la ola rompiente y su clasificación en función de la pendiente de la playa, altura de la ola en aguas profundas y esbeltez de la ola, según el CERC (ref 6)

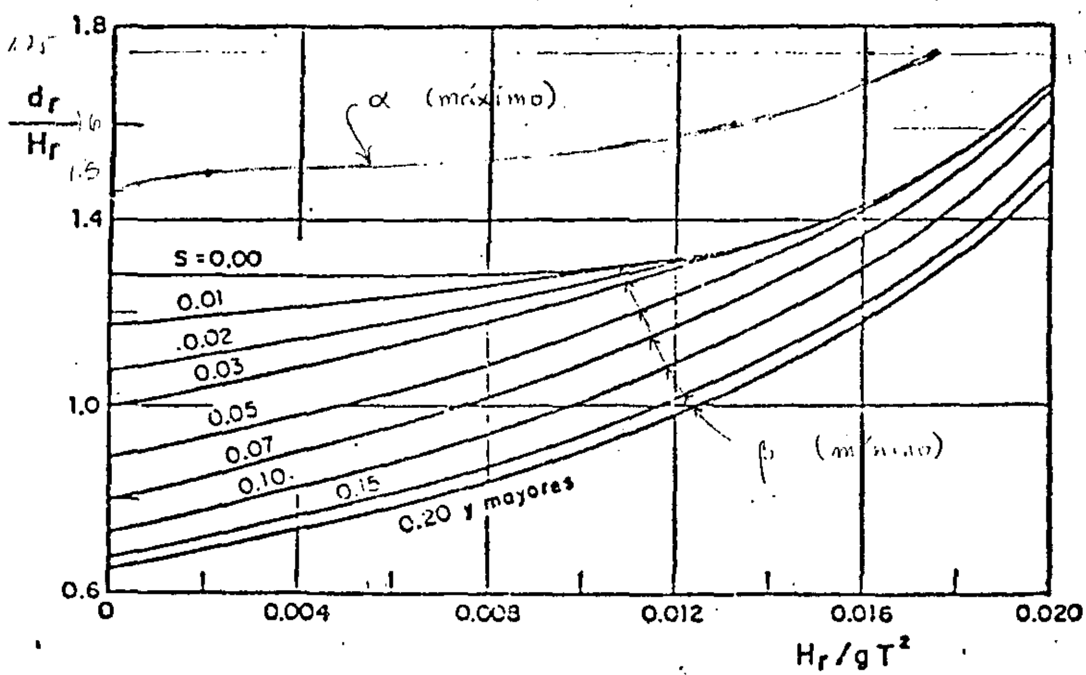
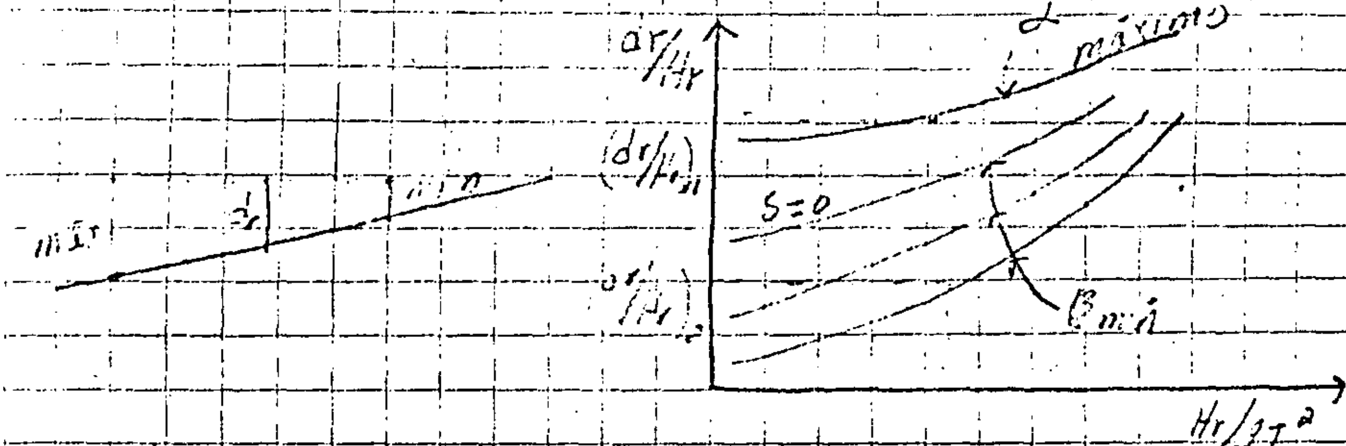


FIG. II.4 Profundidad en rompiente, en función de la pendiente de la playa, altura de la ola rompiente y periodo de la ola, según el CERC (ref 6)

El coeficiente de fondo es $Kd=1$ cuando el frente de onda es paralelo a la batimétrica.

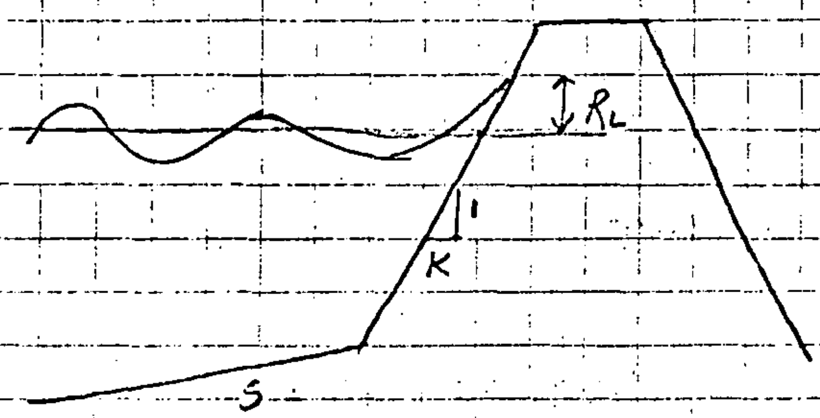


$$\left(\frac{d_r}{H_r}\right)_2 \leq \frac{d_r}{H_r} \leq \left(\frac{d_r}{H_r}\right)_1$$

Para el oleaje en aguas profundas

$$\frac{H_o}{L_o} \leq 0.142 \doteq \frac{1}{7} \quad \text{(rompiente en aguas profundas)}$$

★ Alcance de la ola (run up)



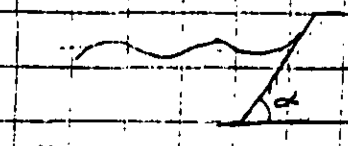
1 La ola rompe antes del llegar a la estructura
2. La ola no rompe antes de llegar a la estructura
2.1 La ola rompe sobre la estructura
2.2 La ola no rompe sobre la estructura.

2.-

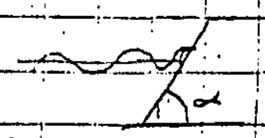
$$R_i = H_o\, D_d\, F\left[\sqrt{\frac{\pi}{2\alpha}} + \pi \frac{H}{L} \coth kd\right]$$

α en radianes = ang tang $1/K$

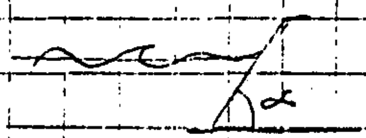
Alcance de la ola (Run-up)

① 

no rompe

② 

la ola rompe sobre la estructura

③ 

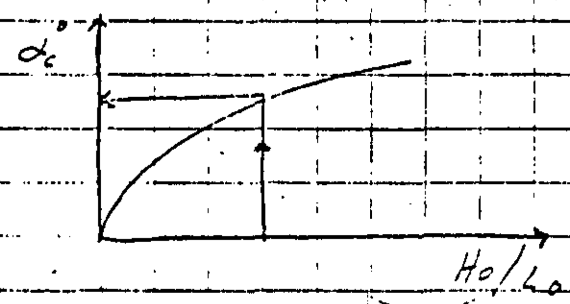
la ola rompe antes de llegar a la estructura

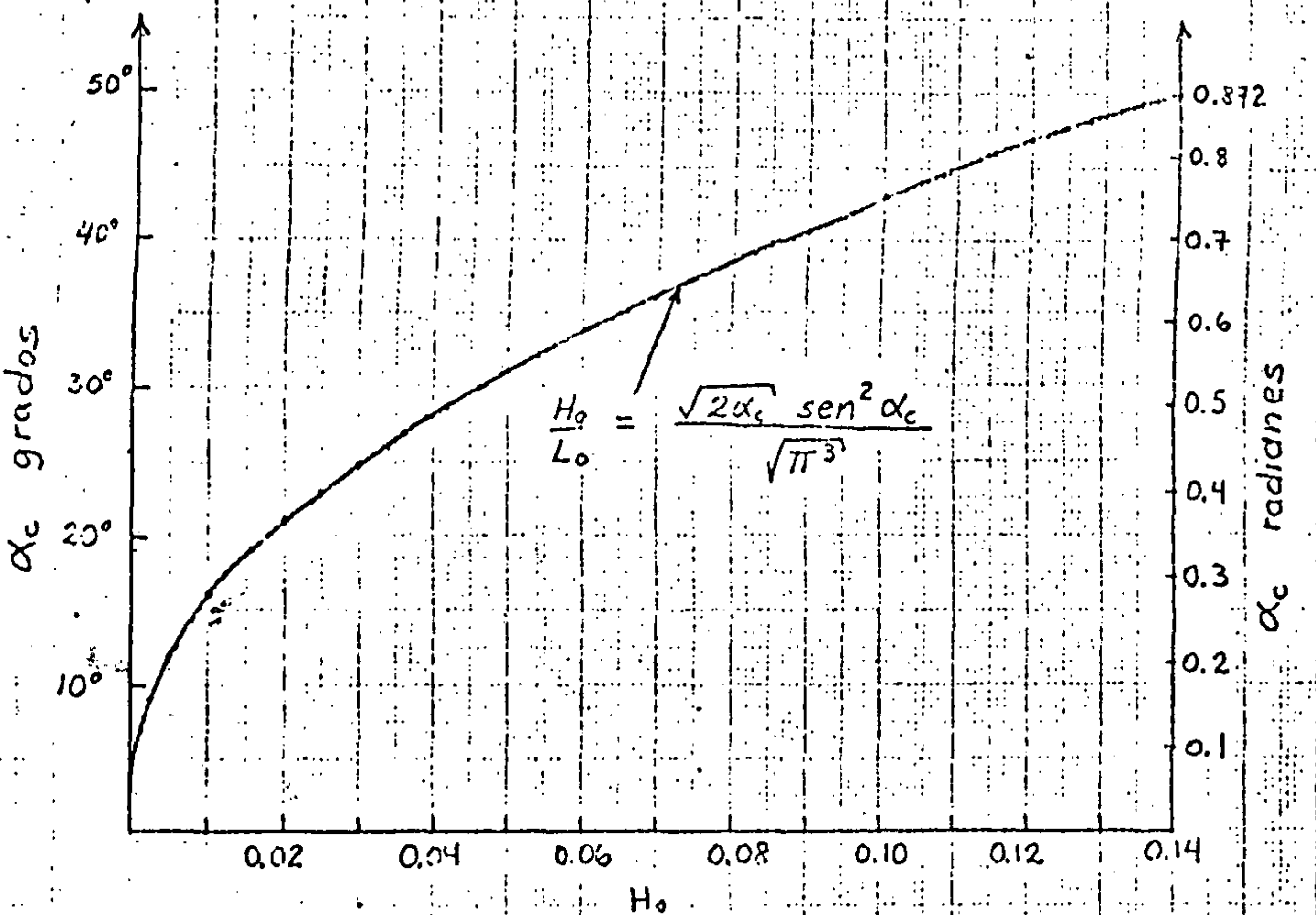
Para los casos ① y ② se utiliza la siguiente expresión

$$\boxed{R_i = H_o\, D_d\, F\left[\sqrt{\frac{\pi}{2\alpha}} + \pi \frac{H}{L}\coth kd\right]}$$

α esta en radianes

para utilizarla es necesario utilizar una gráfica para estimar α_c





Fig

Para obtener el ángulo crítico α_c

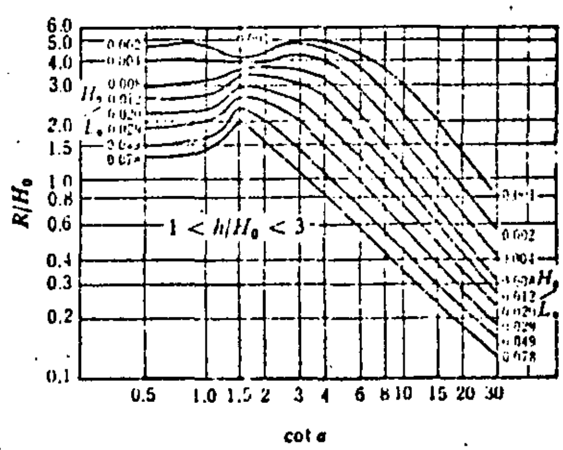


Fig. Wave run-up diagram used for the hypothetical single slope method (after Saville, 1958).

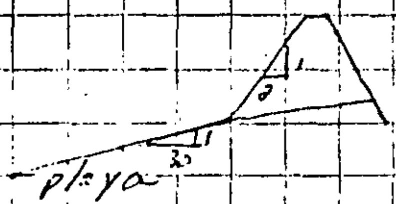
si $\alpha < \alpha_c$ la ola rompe sobre la estructura

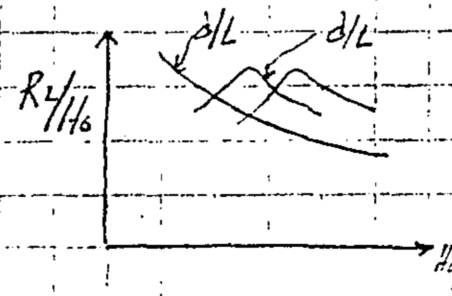
$$y \quad F = \left(\frac{\cot \alpha_c}{\cot \alpha}\right)^{2/3}$$

si $\alpha \geq \alpha_c$ la ola no rompe rompe

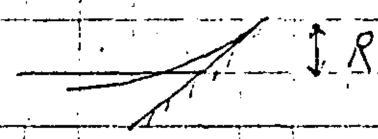
$$y \quad F = 1$$

Para el caso en que la ola rompe antes de llegar a la estructura el alcance de la ola se define a partir de las Fig. 2.10.3



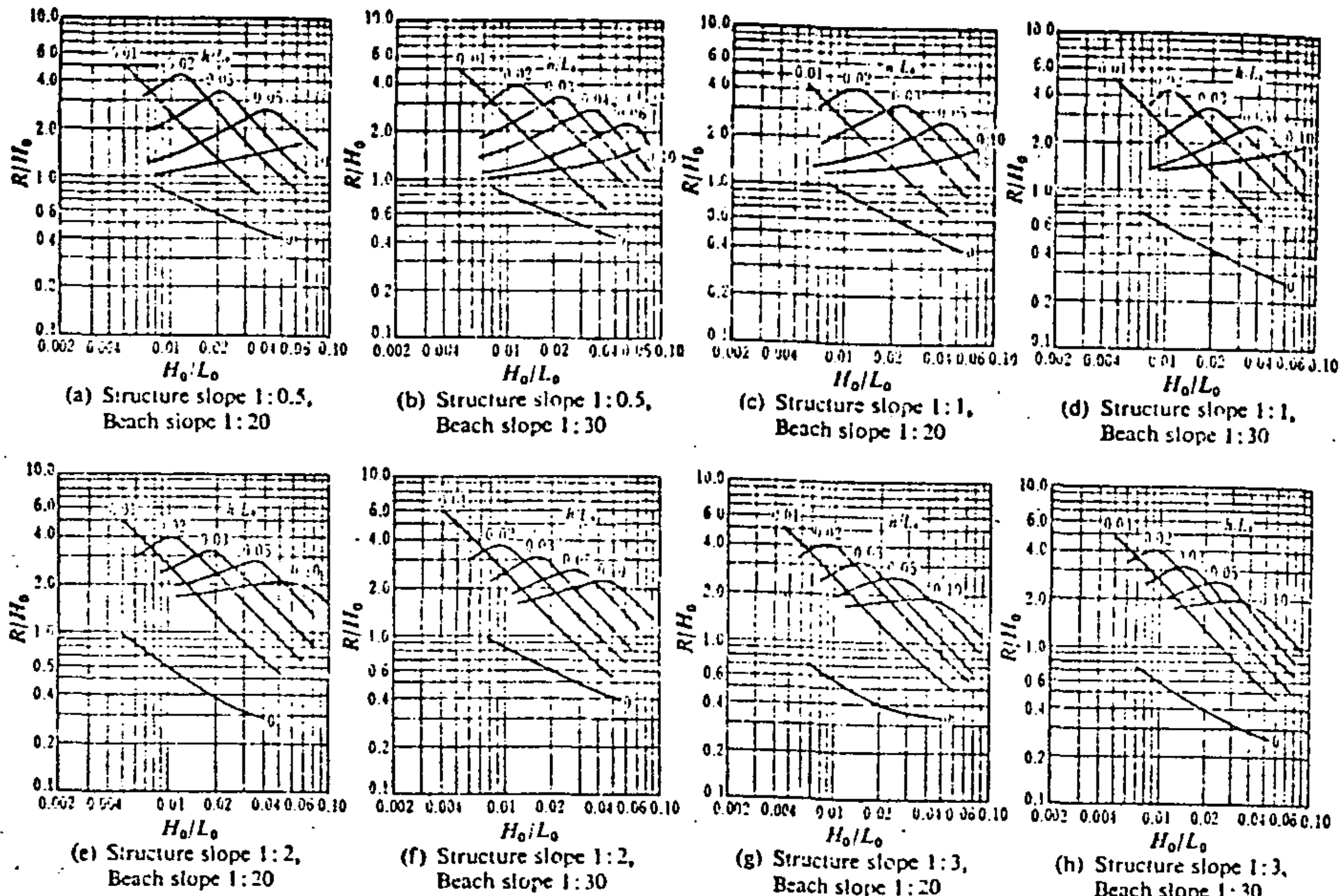


$$\boxed{R = R_L \beta}$$



Es necesario revisar si la ola rompe antes de llegar a la estructura

FIG. 2.10.3



(a) Structure slope 1:0.5, Beach slope 1:20

(b) Structure slope 1:0.5, Beach slope 1:30

(c) Structure slope 1:1, Beach slope 1:20

(d) Structure slope 1:1, Beach slope 1:30

(e) Structure slope 1:2, Beach slope 1:20

(f) Structure slope 1:2, Beach slope 1:30

(g) Structure slope 1:3, Beach slope 1:20

(h) Structure slope 1:3, Beach slope 1:30

• Rompiente antes de llegar a la estructura.

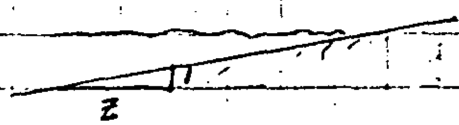
Material y acabado de la superficie	f
Losa de concreto	1-0.9
Pastos	0.85-0.9
una copa de rezoga sobre superf. impermeables	0.80
Piedras acomodadas	0.75-0.80
Piedras redondas	0.60-0.65
" colocados a volteo	0.50-0.60
Dos copas de rezoga	0.50
Elementos artificiales	0.50
Mampostería (dos capas)	0.50-0.55
(una copa)	0.80

★ Alcance de la ola sobre la playa.
(Run-up ó UPRUSH)

$$R = d\,A\,(H/d)^{B}$$

α	5°	10°	15°	25°	45°
A	2.15	3.43	3.75	3.35	3.1
B	0.81	1.04	1.12	1.12	1.15

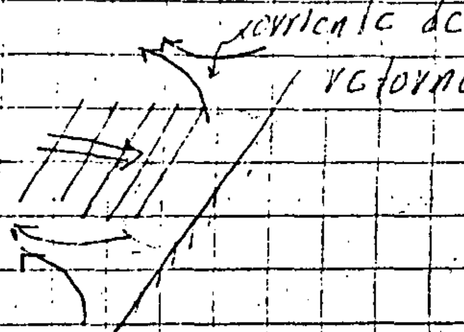
$\alpha = \text{ang tan}\,(1/z)$



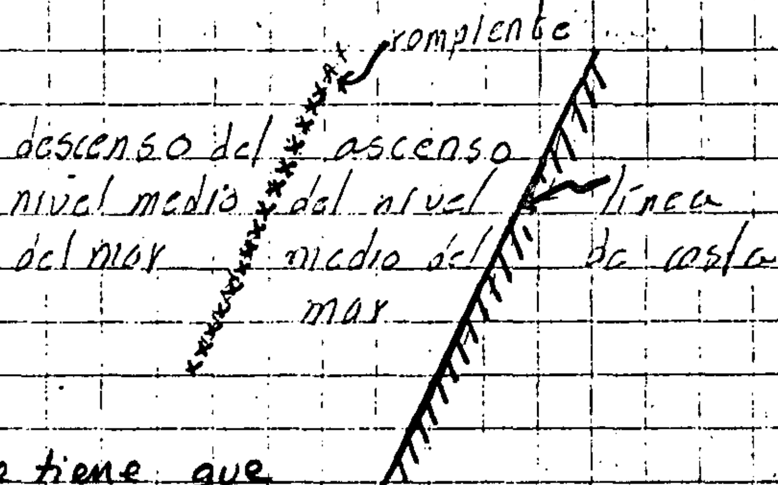
2.6 Ascenso y descenso.

Recordando el concepto de

Transporte de masa

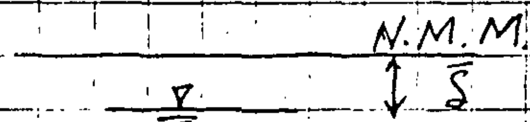


esto se debe a que los partículas sí tienen un desplazamiento en aguas poco profundas.



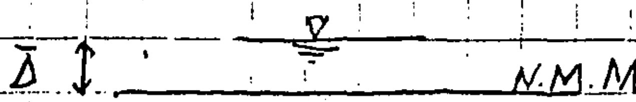
se tiene que

descenso (set-down) $\quad \bar{\delta} = \dfrac{H^2}{8}\,\dfrac{k}{\operatorname{senh} 2kd}$



ascenso (set-up) $\quad \bar{\Delta} = K(d_b - d) + \bar{\delta}_b$

$$; \; K = \frac{1}{1 + (8/3\gamma^2)}$$



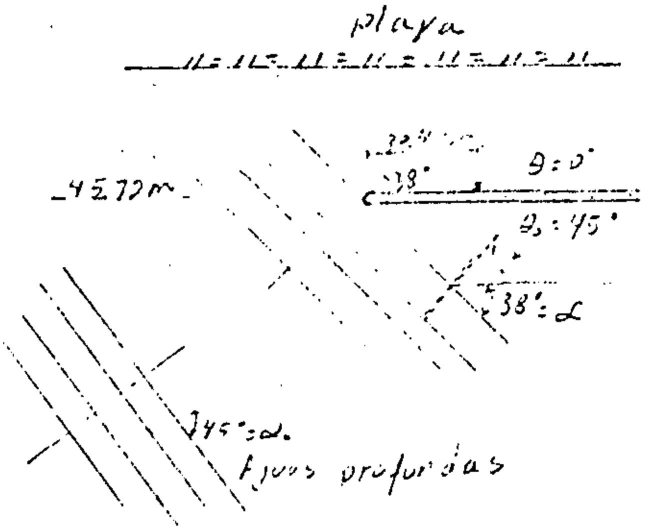
$\gamma = 0.73$

EJEMPLOS

1.- Un oleaje tal que en aguas profundas tiene una altura de 10.85 m y un periodo de 12.6 seg. se aproxima a la línea de playa en una dirección de 45°.

a) En un punto donde la profundidad es 45.72 m las olas han sido afectadas, ¿cuál es la altura de ola en ese punto?

b) Si en ese punto ellas encuentran un rompeolas que es paralelo a la línea de playa ¿cuál será la altura del oleaje en el lado protegido a 30.48 m desde el extremo del rompeolas sobre este.



Datos: $H_0 = 10.85$ m, $T = 12.6$ seg

$$L_0 = \frac{gT^2}{2\pi} = \frac{9.81(12.6)^2}{2\pi} = 247.87 m$$

$\frac{d_0}{L_0} = 0.5$; $d_0 = 0.5(247.87) = 123.94$ m

para $d = 45.72$ m

$$\frac{d}{L_0} = \frac{45.72}{247.87} = 0.1844$$

$\frac{d}{L} = 0.210$ $\quad L = 215.66$ m

de tablas $D_d = 0.9151$ (coef. de fondo)

$$C_0 = \frac{L_0}{T_0} = \frac{247.87}{12.6} = 19.67 \text{ m/seg};$$

$$C = \frac{L}{T} = \frac{215.66}{12.6} = 17.16 \text{ m/seg}$$

$$\alpha = \text{ang sen}\left(\frac{C}{C_0} \operatorname{sen}\alpha_0\right)$$

$$\alpha = \text{ang sen}\left(\frac{17.16}{19.67} \operatorname{sen} 45\right)$$

$\alpha = 38.088°$

$\alpha_0 = 45°$

$$K_D = \sqrt{\frac{\cos \theta_0}{\cos \alpha}} = \sqrt{\frac{\cos 45}{\cos 38.088}} = 0.9478 \quad (\text{coef. de refracción})$$

$$\therefore H = H_0 K_D D_d = (10.85)(0.9151)(0.9478)$$

$$\therefore \boxed{H = 9.41\ m}$$ ✓

b) Datos $\theta_0 = 38°$; $\theta = 0°$; $\frac{r}{L} = \frac{30.48}{215.66} = 0.1413$

de la fig $K_D = 0.65$ (coef. de difracción)

$$H = K_D H_i = (0.65)(9.41)$$

$$\therefore \boxed{H = 6.12\ m}$$ ✗

2. En aguas profundas el oleaje tiene una altura de 5 m y una longitud de 120 m se acerca perpendicularmente a la línea de costa. El fondo marino tiene una pendiente de 0.05. Encontrar la altura rompiente y la profundidad de la rompiente, y la distancia a partir de la línea de costa donde se presenta la rompiente.

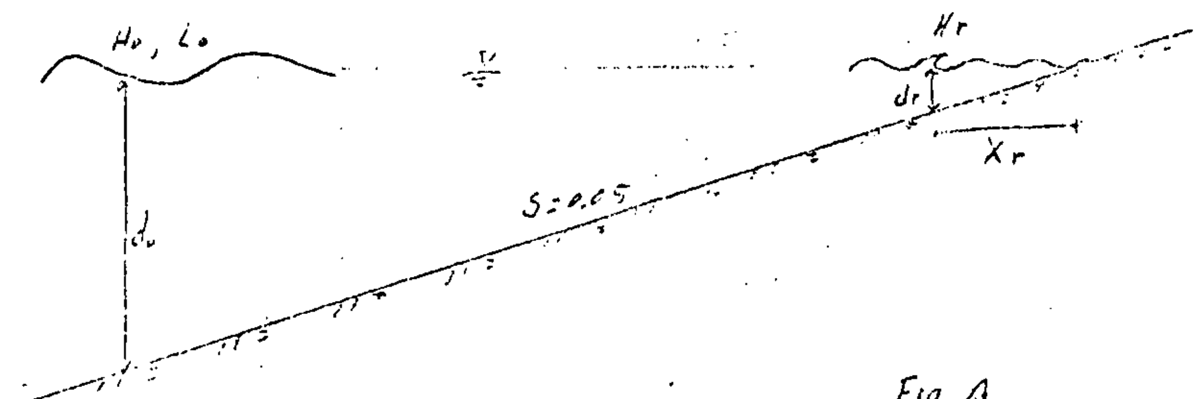


Fig. A

Datos: $H_o = 5\,m$, $L_o = 120\,m$, $S = 0.05$, $\alpha_o = 0°$

se propone $d_r = 6.27\,m$ (de la fig A $\alpha = 0° = \alpha_o$)

$H_o' = H_o K_d$ ($K_d = 1$ puesto que $\alpha = \alpha_o$; $b = b_o$)

$H_o' = 5(1) = 5\,m$; $\frac{H_o'}{L_o} = \frac{5}{120} = 0.0417$

con $\frac{H_o'}{L_o} = 0.0417$ y $S = 0.05$ de la fig. II.3 se obtiene

$\frac{H_r}{H_o'} = 1.17$; $H_r = 1.17 H_o' = 1.17(5) = 5.85\,m$

Revisión $T = \sqrt{2\pi L_o / g} = \sqrt{2\pi(120)/9.81} = 8.77\,seg$

como $\frac{H_r}{gT^2} = \frac{5.85}{9.81(8.77)^2} = 0.0078$

con $\frac{H_r}{gT^2} = 0.0078$ y $S = 0.05$, de la figura se obtiene

$\frac{d_r}{H_r} \doteq 1.07$; $d_r = 1.07 H_r = 1.07(5.85) = 6.26\,m$

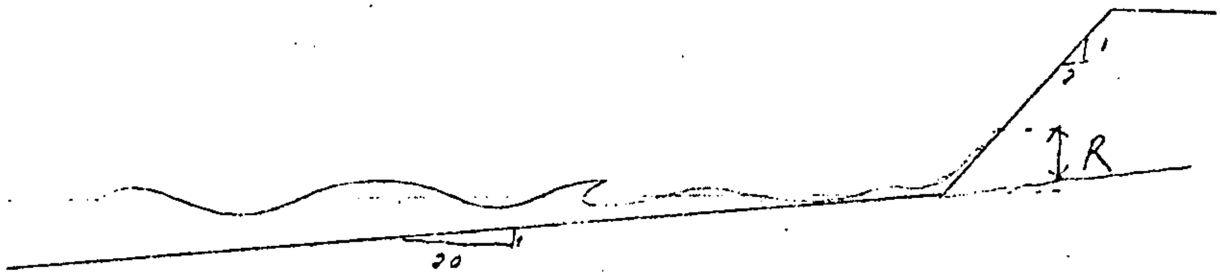
$\therefore$ $\boxed{H_r = 5.85\,m}$ ✓

$\boxed{d_r = 6.26\,m}$ ✓

$\tan\phi = 0.05 = \frac{d_r}{X_r}$; $X_r = \frac{d_r}{0.05} = \frac{6.26}{0.05}$

$\therefore$ $\boxed{X_r = 125.20\,m}$ ✓

3. Determinar el alcance de la ola sobre un muro de piedra redonda que tiene un talúd de 2:1; si el oleaje a una profundidad de 40 m tiene una altura de 5 m y un periodo de 10 seg.



$L_o = \frac{gT^2}{2\pi} = \frac{9.81(10)^2}{2\pi} = 156.13$ m ; $\frac{d_o}{L_o} = 0.5 \Rightarrow d_o = 78.06$ m

$\therefore H = 5$ m pertenece a la condición de aguas intermedias

$\frac{d}{L_o} = \frac{40}{156.13} = 0.2562 \Rightarrow \frac{d}{L} = 0.2733$ y $L = 146.36$ m

y también $D_d = 0.9344$ (coeficiente de fondo)

se supone que $\alpha = \alpha_o$ y $K_d = 1$ (coef. refracción)

$H = H_o D_d K_d$; $H_o = \frac{H}{D_d K_d} = \frac{5}{0.9344} = 5.35$ m

$\therefore$ Las condiciones de aguas profundas son $\begin{cases} H_o = 5.35\,m \\ T_o = 10\,seg \end{cases}$

A continuación se calculan las condiciones de oleaje rompiente

$H_o' = H_o K_d = (5.35)(1) = 5.35\ m$

$\frac{H_o'}{L_o} = \frac{5.35}{156.13} = 0.034$ y como $S = 0.05$

de la fig. II.3 se estima $\frac{H_r}{H_o'} = 1.23$

$H_r = 1.23 H_o' = 1.23(5.35) = 6.58\ m$

Por otro lado

$\frac{H_r}{gT^2} = \frac{6.58}{9.81(10)^2} = 0.0067$ de la fig. II.4 y con $S = 0.05$

se estima $\frac{d_r}{H_r} = 1.04$; $d_r = 1.04(6.58) = 6.88\ m$

∴ Las condiciones de oleaje rompiente son

$H_r = 6.58\ m$

$d_r = 6.88\ m$

y la ola rompe antes de llegar a la estructura

de la fig. 2.10.3 con $S = 0.05$ y $I = 2$ se estima

para $\frac{h}{L_o} = 0$ y $\frac{H_o}{L_o} = \frac{5.35}{156.13} = 0.0343$

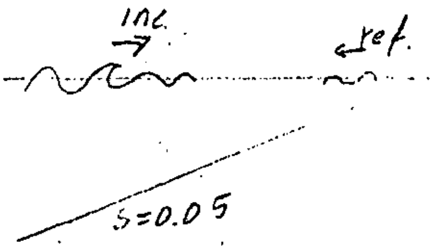
$\frac{R_L}{H_o} = 0.3$; $R_L = 0.3(5.35) = 1.6\ m$ y $f = 0.65$

como $R = f R_L = (0.65)(1.6)$ $\boxed{R \doteq 1\ m}$

4. ¿Cual será la altura del oleaje reflejada desde una playa lisa e impermeable con talud 20:1, si el oleaje en aguas profundas tiene una altura de 6 m y un periodo de 12 seg

Datos: $H_0 = 6\,m$ $\quad T = 12\,seg$

$L_0 = \frac{gT^2}{2\pi} = \frac{9.81(12)^2}{2\pi} = 224.83\,m$



se supone $K_d = 1$

$H_0' = H_0 K_d = 6\,m \quad ; \frac{H_0'}{L_0} = \frac{6}{224.83} = 0.0267$

de la fig. II.3 $\quad \frac{H_r}{H_0'} = 1.29 \qquad H_r = 1.29(6) = 7.74\,m$

Con $\frac{H_r}{gT^2} = \frac{7.74}{9.81(12)^2} = 0.0055$ de la fig. II.4

se estima $\frac{d_r}{H_r} = 1.032$; $d_r = 1.032(7.74) \doteq 8\,m$

∴ Las condiciones de oleaje rompiente son

$H_r = 7.74\,m$

$d_r = 8\,m$

El coeficiente de reflexión es

$K_r = \chi_1 \cdot \chi_2$

$\chi_1 = 0.8$ (playa lisa e imp.)

$$\left(\frac{H_0}{L_0}\right)_{máx} = \left(\frac{2\theta}{\pi}\right)^{1/2} \frac{sen^2\theta}{\pi}$$

$\theta = ang.\ tang(1/20) = 2.8624° = 0.05\ rad$

$$\left(\frac{H_0}{L_0}\right)_{máx} = \left[\frac{2(0.05)}{\pi}\right]^{1/2} \frac{sen^2 2.8624}{\pi} = 0.0001$$

$$\chi_2 = \frac{(H_0/L_0)_{máx}}{(H_0/L_0)} = \frac{0.0001}{(6/224.85)} = 0.0053$$

$K_r = \chi_1 \cdot \chi_2 = (0.8)(0.0053) = 0.0042$ ✓

$a_{ref} = K_r\, a_{inc}$; $a_{inc} = \frac{1}{2} H_r = \frac{1}{2}(7.74) = 3.87$

$a_{ref} = (0.0042)(3.87) = 0.016\ m$

$\therefore\ H_{ref} = 2a_{ref}$ $\boxed{H_{ref} = 0.033\ m}$

5. Defina refracción, difracción y reflexión.

<u>Refracción</u>: Se define como el proceso mediante el cual, las ondas cambián su dirección de propagación a causa de una disminución de la profundidad.

Difracción: Se llama así al proceso en el que la energía de una onda, decae lateralmente debido a la falta de soporpo generado al interceptar la esquina de algún obstaculo.

Reflexión. Ocurre por el choque de una onda con algún obstaculo, lo cual da origén a otra nueva onda que parte del obstáculo.

APPENDIX 1. TABLE OF FUNCTIONS OF d/L_0

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
0	0	0	0	0	1	1	0	0	1	1	0		
.0001000	.003990	.02507	.02506	.02507	1.0003	.9997	.05014	.05016	1.001	.9998	.02506	4.467	7,855
.0002000	.005643	.03546	.03544	.03547	1.0006	.9994	.07091	.07097	1.003	.9996	.03543	3.757	3,928
.0003000	.006912	.04343	.04340	.04344	1.0009	.9991	.08686	.08697	1.004	.9994	.04338	3.395	2,620
.0004000	.007982	.05015	.05011	.05018	1.0013	.9987	.1003	.1005	1.005	.9992	.05007	3.160	1,965
.0005000	.008925	.05608	.05602	.05611	1.0016	.9984	.1122	.1124	1.006	.9990	.05596	2.989	1,572
.0006000	.009778	.06144	.06136	.06148	1.0019	.9981	.1229	.1232	1.008	.9988	.06128	2.856	1,311
.0007000	.01056	.06637	.06627	.06642	1.0022	.9978	.1327	.1331	1.009	.9985	.06617	2.749	1,124
.0008000	.01129	.07096	.07084	.07102	1.0025	.9975	.1419	.1424	1.010	.9983	.07072	2.659	983.5
.0009000	.01198	.07527	.07513	.07534	1.0028	.9972	.1505	.1511	1.011	.9981	.07499	2.582	874.3
.001000	.01263	.07935	.07918	.07943	1.0032	.9969	.1587	.1594	1.013	.9979	.07902	2.515	787.0
.001100	.01325	.08323	.08304	.08333	1.0035	.9966	.1665	.1672	1.014	.9977	.08285	2.456	715.6
.001200	.01384	.08694	.08672	.08705	1.0038	.9962	.1739	.1748	1.015	.9975	.08651	2.404	656.1
.001300	.01440	.09050	.09026	.09063	1.0041	.9959	.1810	.1820	1.016	.9973	.09001	2.357	605.8
.001400	.01495	.09393	.09365	.09407	1.0044	.9956	.1879	.1890	1.018	.9971	.09338	2.314	562.6
.001500	.01548	.09723	.09693	.09739	1.0047	.9953	.1945	.1957	1.019	.9969	.09663	2.275	525
.001600	.01598	.1004	.1001	.1006	1.0051	.9949	.2009	.2022	1.020	.9967	.09977	2.239	493
.001700	.01648	.1035	.1032	.1037	1.0054	.9946	.2071	.2086	1.022	.9965	.1028	2.205	463
.001800	.01696	.1066	.1062	.1068	1.0057	.9943	.2131	.2147	1.023	.9962	.1058	2.174	438
.001900	.01743	.1095	.1091	.1097	1.0060	.9940	.2190	.2207	1.024	.9960	.1087	2.145	415
.002000	.01788	.1123	.1119	.1125	1.0063	.9937	.2247	.2266	1.025	.9958	.1114	2.119	394
.002100	.01832	.1151	.1146	.1154	1.0066	.9934	.2303	.2323	1.027	.9956	.1141	2.094	376
.002200	.01876	.1178	.1173	.1181	1.0069	.9931	.2357	.2379	1.028	.9954	.1167	2.070	359
.002300	.01918	.1205	.1199	.1208	1.0073	.9928	.2410	.2433	1.029	.9952	.1193	2.047	343
.002400	.01959	.1231	.1225	.1234	1.0076	.9925	.2462	.2487	1.031	.9950	.1219	2.025	329
.002500	.02000	.1257	.1250	.1260	1.0079	.9922	.2513	.2540	1.032	.9948	.1243	2.005	316
.002600	.02040	.1282	.1275	.1285	1.0082	.9919	.2563	.2592	1.033	.9946	.1268	1.986	304
.002700	.02079	.1306	.1299	.1310	1.0085	.9916	.2612	.2642	1.034	.9944	.1292	1.967	292
.002800	.02117	.1330	.1323	.1334	1.0089	.9912	.2661	.2692	1.036	.9942	.1315	1.950	282
.002900	.02155	.1354	.1346	.1358	1.0092	.9909	.2708	.2741	1.037	.9939	.1338	1.933	272
.003000	.02192	.1377	.1369	.1382	1.0095	.9906	.2755	.2790	1.038	.9937	.1360	1.917	263
.003100	.02228	.1400	.1391	.1405	1.0098	.9903	.2800	.2837	1.040	.9935	.1382	1.902	255
.003200	.02264	.1423	.1413	.1427	1.0101	.9900	.2845	.2884	1.041	.9933	.1404	1.887	247
.003300	.02300	.1445	.1435	.1449	1.0104	.9897	.2890	.2930	1.042	.9931	.1425	1.873	240
.003400	.02335	.1467	.1456	.1472	1.0108	.9893	.2934	.2976	1.043	.9929	.1446	1.860	233
.003500	.02369	.1488	.1477	.1494	1.0111	.9890	.2977	.3021	1.045	.9927	.1466	1.847	226
.003600	.02403	.1510	.1498	.1515	1.0114	.9887	.3020	.3065	1.046	.9925	.1487	1.834	220
.003700	.02436	.1531	.1519	.1537	1.0117	.9884	.3061	.3109	1.047	.9923	.1507	1.822	214
.003800	.02469	.1551	.1539	.1558	1.0121	.9881	.3103	.3153	1.049	.9921	.1527	1.810	208
.003900	.02502	.1572	.1559	.1579	1.0124	.9878	.3144	.3196	1.050	.9919	.1546	1.799	203
.004000	.02534	.1592	.1579	.1599	1.0127	.9875	.3184	.3238	1.051	.9917	.1565	1.788	198
.004100	.02566	.1612	.1598	.1619	1.0130	.9872	.3224	.3280	1.052	.9915	.1584	1.777	193
.004200	.02597	.1632	.1617	.1639	1.0133	.9869	.3263	.3322	1.054	.9912	.1602	1.767	189
.004300	.02628	.1651	.1636	.1659	1.0137	.9865	.3302	.3362	1.055	.9910	.1621	1.756	184
.004400	.02659	.1671	.1655	.1678	1.0140	.9862	.3341	.3403	1.056	.9908	.1640	1.746	180
.004500	.02689	.1690	.1674	.1698	1.0143	.9859	.3380	.3444	1.058	.9906	.1658	1.737	176
.004600	.02719	.1708	.1692	.1717	1.0146	.9856	.3417	.3483	1.059	.9904	.1676	1.727	172
.004700	.02749	.1727	.1710	.1736	1.0149	.9853	.3454	.3523	1.060	.9902	.1693	1.718	169
.004800	.02778	.1745	.1728	.1754	1.0153	.9849	.3491	.3562	1.062	.9900	.1711	1.709	165
.004900	.02807	.1764	.1746	.1773	1.0156	.9846	.3527	.3601	1.063	.9898	.1728	1.701	162
.005000	.02836	.1782	.1764	.1791	1.0159	.9843	.3564	.3640	1.064	.9896	.1746	1.692	159
.005100	.02864	.1800	.1781	.1809	1.0162	.9840	.3599	.3678	1.066	.9894	.1762	1.684	156
.005200	.02893	.1818	.1798	.1827	1.0166	.9837	.3635	.3715	1.067	.9892	.1779	1.676	153
.005300	.02921	.1835	.1815	.1845	1.0169	.9834	.3670	.3753	1.068	.9889	.1795	1.669	150
.005400	.02948	.1852	.1832	.1863	1.0172	.9831	.3705	.3790	1.069	.9887	.1811	1.662	147
.005500	.02976	.1870	.1848	.1880	1.0175	.9828	.3739	.3827	1.071	.9885	.1827	1.654	145
.005600	.03003	.1887	.1865	.1898	1.0178	.9825	.3774	.3864	1.072	.9883	.1843	1.647	142
.005700	.03030	.1904	.1881	.1915	1.0182	.9822	.3808	.3900	1.073	.9881	.1859	1.640	140
.005800	.03057	.1921	.1897	.1932	1.0185	.9818	.3841	.3937	1.075	.9879	.1874	1.633	137
.005900	.03083	.1937	.1913	.1949	1.0188	.9815	.3875	.3972	1.076	.9877	.1890	1.626	135
.006000	.03110	.1954	.1929	.1967	1.0192	.9812	.3908	.4008	1.077	.9875	.1905	1.620	133
.006100	.03136	.1970	.1945	.1983	1.0195	.9809	.3941	.4044	1.079	.9873	.1920	1.614	130
.006200	.03162	.1987	.1961	.2000	1.0198	.9806	.3973	.4079	1.080	.9871	.1935	1.607	128
.006300	.03188	.2003	.1976	.2016	1.0201	.9803	.4006	.4114	1.081	.9869	.1950	1.601	126
.006400	.03213	.2019	.1992	.2033	1.0205	.9799	.4038	.4148	1.083	.9867	.1965	1.595	124
.006500	.03238	.2035	.2007	.2049	1.0208	.9796	.4070	.4183	1.084	.9865	.1980	1.589	123
.006600	.03264	.2051	.2022	.2065	1.0211	.9793	.4101	.4217	1.085	.9863	.1994	1.583	121
.006700	.03289	.2066	.2037	.2081	1.0214	.9790	.4133	.4251	1.087	.9860	.2009	1.578	119
.006800	.03313	.2082	.2052	.2097	1.0217	.9787	.4164	.4285	1.088	.9858	.2023	1.572	117
.006900	.03338	.2097	.2067	.2113	1.0221	.9784	.4195	.4319	1.089	.9856	.2037	1.567	116

APPENDIX I. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
.007000	.03362	.2113	.2082	.2128	1.0224	.9781	.4225	.4352	1.091	.9854	.2051	1.561	114
.007100	.03387	.2128	.2096	.2144	1.0227	.9778	.4256	.4386	1.092	.9852	.2065	1.556	112
.007200	.03411	.2143	.2111	.2160	1.0231	.9774	.4286	.4419	1.093	.9850	.2079	1.551	111
.007300	.03435	.2158	.2125	.2175	1.0234	.9771	.4316	.4452	1.095	.9848	.2093	1.546	109
.007400	.03459	.2173	.2139	.2190	1.0237	.9768	.4346	.4484	1.096	.9846	.2106	1.541	108
.007500	.03482	.2188	.2154	.2205	1.0240	.9765	.4376	.4517	1.097	.9844	.2120	1.536	106
.007600	.03506	.2203	.2168	.2221	1.0244	.9762	.4406	.4549	1.099	.9842	.2134	1.531	105
.007700	.03529	.2218	.2182	.2236	1.0247	.9759	.4435	.4582	1.100	.9840	.2147	1.526	104
.007800	.03552	.2232	.2196	.2251	1.0250	.9756	.4464	.4614	1.101	.9838	.2160	1.521	102
.007900	.03576	.2247	.2209	.2265	1.0253	.9753	.4493	.4646	1.103	.9836	.2173	1.517	101
.008000	.03598	.2261	.2223	.2280	1.0257	.9750	.4522	.4678	1.104	.9834	.2186	1.512	100
.008100	.03621	.2275	.2237	.2295	1.0260	.9747	.4551	.4709	1.105	.9832	.2199	1.508	98.6
.008200	.03644	.2290	.2250	.2310	1.0263	.9744	.4579	.4741	1.107	.9830	.2212	1.503	97.5
.008300	.03666	.2304	.2264	.2324	1.0266	.9741	.4607	.4772	1.108	.9827	.2225	1.499	96.3
.008400	.03689	.2318	.2277	.2338	1.0270	.9737	.4636	.4803	1.109	.9825	.2237	1.495	95.2
.008500	.03711	.2332	.2290	.2353	1.0273	.9734	.4664	.4834	1.111	.9823	.2250	1.491	94.1
.008600	.03733	.2346	.2303	.2367	1.0276	.9731	.4691	.4865	1.112	.9821	.2262	1.487	93.0
.008700	.03755	.2360	.2317	.2381	1.0280	.9728	.4719	.4896	1.113	.9819	.2275	1.482	91.9
.008800	.03777	.2373	.2330	.2396	1.0283	.9725	.4747	.4927	1.115	.9817	.2287	1.478	90.9
.008900	.03799	.2387	.2343	.2410	1.0286	.9722	.4774	.4957	1.116	.9815	.2300	1.474	89.9
.009000	.03821	.2401	.2356	.2424	1.0290	.9718	.4801	.4988	1.118	.9813	.2312	1.471	88.9
.009100	.03842	.2414	.2368	.2438	1.0293	.9715	.4828	.5018	1.119	.9811	.2324	1.467	88.0
.009200	.03864	.2428	.2381	.2452	1.0296	.9712	.4855	.5049	1.120	.9809	.2336	1.463	87.1
.009300	.03885	.2441	.2394	.2465	1.0299	.9709	.4882	.5079	1.122	.9807	.2348	1.459	86.1
.009400	.03906	.2455	.2407	.2479	1.0303	.9706	.4909	.5109	1.123	.9805	.2360	1.456	85.2
.009500	.03928	.2468	.2419	.2493	1.0306	.9703	.4936	.5138	1.124	.9803	.2371	1.452	84.3
.009600	.03949	.2481	.2431	.2507	1.0309	.9700	.4962	.5168	1.126	.9801	.2383	1.448	83.5
.009700	.03970	.2494	.2443	.2520	1.0313	.9697	.4988	.5198	1.127	.9799	.2394	1.445	82.7
.009800	.03990	.2507	.2456	.2534	1.0316	.9694	.5014	.5227	1.128	.9797	.2406	1.442	81.8
.009900	.04011	.2520	.2468	.2547	1.0319	.9691	.5040	.5257	1.130	.9794	.2417	1.438	81.0
.01000	.04032	.2533	.2480	.2560	1.0322	.9688	.5066	.5286	1.131	.9792	.2429	1.435	80.2
.01100	.04233	.2660	.2598	.2691	1.0356	.9656	.5319	.5574	1.145	.9772	.2539	1.403	73.1
.01200	.04426	.2781	.2711	.2817	1.0389	.9625	.5562	.5853	1.159	.9751	.2643	1.375	67.1
.01300	.04612	.2898	.2820	.2938	1.0423	.9594	.5795	.6125	1.173	.9731	.2743	1.350	62.1
.01400	.04791	.3010	.2924	.3056	1.0456	.9564	.6020	.6391	1.187	.9710	.2838	1.327	57.8
.01500	.04964	.3119	.3022	.3170	1.0490	.9533	.6238	.6651	1.201	.9690	.2928	1.307	54.0
.01600	.05132	.3225	.3117	.3281	1.0524	.9502	.6450	.6906	1.215	.9670	.3014	1.288	50.8
.01700	.05296	.3328	.3209	.3389	1.0559	.9471	.6655	.7158	1.230	.9649	.3096	1.271	47.9
.01800	.05455	.3428	.3298	.3495	1.0593	.9440	.6856	.7405	1.244	.9629	.3176	1.255	45.3
.01900	.05611	.3525	.3386	.3599	1.0628	.9409	.7051	.7650	1.259	.9609	.3253	1.240	43.0
.02000	.05763	.3621	.3470	.3701	1.0663	.9378	.7242	.7891	1.274	.9588	.3327	1.226	41.0
.02100	.05912	.3714	.3552	.3800	1.0698	.9348	.7429	.8131	1.289	.9568	.3399	1.213	39.1
.02200	.06057	.3806	.3632	.3898	1.0733	.9317	.7612	.8368	1.304	.9548	.3468	1.201	37.4
.02300	.06200	.3896	.3710	.3995	1.0768	.9287	.7791	.8603	1.319	.9528	.3535	1.189	35.9
.02400	.06340	.3984	.3786	.4090	1.0804	.9256	.7967	.8837	1.335	.9508	.3600	1.178	34.4
.02500	.06478	.4070	.3860	.4184	1.0840	.9225	.8140	.9069	1.350	.9488	.3662	1.168	33.1
.02600	.06613	.4155	.3932	.4276	1.0876	.9195	.8310	.9310	1.366	.9468	.3722	1.159	31.9
.02700	.06747	.4239	.4002	.4367	1.0912	.9164	.8478	.9530	1.381	.9448	.3781	1.150	30.8
.02800	.06878	.4322	.4071	.4457	1.0949	.9133	.8643	.9760	1.397	.9428	.3838	1.141	29.8
.02900	.07007	.4403	.4138	.4546	1.0985	.9103	.8805	.9988	1.413	.9408	.3893	1.133	28.8
.03000	.07135	.4483	.4205	.4634	1.1021	.9073	.8966	1.022	1.430	.9388	.3947	1.125	27.9
.03100	.07260	.4562	.4269	.4721	1.1059	.9042	.9124	1.044	1.446	.9369	.4000	1.118	27.1
.03200	.07385	.4640	.4333	.4808	1.1096	.9012	.9280	1.067	1.462	.9349	.4051	1.111	26.3
.03300	.07507	.4717	.4395	.4894	1.1133	.8982	.9434	1.090	1.479	.9329	.4100	1.104	25.6
.03400	.07630	.4794	.4457	.4980	1.1171	.8952	.9588	1.113	1.496	.9309	.4149	1.098	24.8
.03500	.07748	.4868	.4517	.5064	1.1209	.8921	.9737	1.135	1.513	.9289	.4196	1.092	24.19
.03600	.07867	.4943	.4577	.5147	1.1247	.8891	.9886	1.158	1.530	.9270	.4242	1.086	23.56
.03700	.07984	.5017	.4635	.5230	1.1285	.8861	1.0033	1.180	1.547	.9250	.4287	1.080	22.97
.03800	.08100	.5090	.4691	.5312	1.1324	.8831	1.018	1.203	1.564	.9230	.4330	1.075	22.42
.03900	.08215	.5162	.4747	.5394	1.1362	.8801	1.032	1.226	1.582	.9211	.4372	1.069	21.90
.04000	.08329	.5233	.4802	.5475	1.1401	.8771	1.047	1.248	1.600	.9192	.4414	1.064	21.40
.04100	.08442	.5304	.4857	.5556	1.1440	.8741	1.061	1.271	1.617	.9172	.4455	1.059	20.92
.04200	.08553	.5374	.4911	.5637	1.1479	.8711	1.075	1.294	1.636	.9153	.4495	1.055	20.46
.04300	.08664	.5444	.4964	.5717	1.1518	.8681	1.089	1.317	1.654	.9133	.4534	1.050	20.03
.04400	.08774	.5513	.5015	.5796	1.1558	.8652	1.103	1.340	1.672	.9114	.4571	1.046	19.62
.04500	.08883	.5581	.5066	.5876	1.1599	.8621	1.116	1.363	1.691	.9095	.4607	1.042	19.23
.04600	.08991	.5649	.5116	.5954	1.1639	.8592	1.130	1.386	1.709	.9076	.4643	1.038	18.85
.04700	.09098	.5717	.5166	.6033	1.1679	.8562	1.143	1.409	1.728	.9057	.4679	1.034	18.49
.04800	.09205	.5784	.5215	.6111	1.1720	.8532	1.157	1.433	1.747	.9037	.4713	1.030	18.15
.04900	.09311	.5850	.5263	.6189	1.1760	.8503	1.170	1.456	1.766	.9018	.4746	1.026	17.82

APPENDIX I. (Continued)

d/L_o	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.05000	.09416	.5916	.5310	.6267	1.1802	.8473	1.183	1.479	1.786	.8999	.4779	1.023	17.50
.05100	.09520	.5981	.5357	.6344	1.1843	.8444	1.196	1.503	1.805	.8980	.4811	1.019	17.19
.05200	.09623	.6046	.5403	.6421	1.1884	.8415	1.209	1.526	1.825	.8961	.4842	1.016	16.90
.05300	.09726	.6111	.5449	.6499	1.1926	.8385	1.222	1.550	1.845	.8943	.4873	1.013	16.62
.05400	.09829	.6176	.5494	.6575	1.1968	.8356	1.235	1.574	1.865	.8924	.4903	1.010	16.35
.05500	.09930	.6239	.5538	.6652	1.2011	.8326	1.248	1.598	1.885	.8905	.4932	1.007	16.09
.05600	.1003	.6303	.5582	.6729	1.2053	.8297	1.261	1.622	1.906	.8886	.4960	1.004	15.84
.05700	.1013	.6366	.5626	.6805	1.2096	.8267	1.273	1.646	1.926	.8867	.4988	1.001	15.60
.05800	.1023	.6428	.5668	.6880	1.2138	.8239	1.286	1.670	1.947	.8849	.5015	.9985	15.36
.05900	.1033	.6491	.5711	.6956	1.2181	.8209	1.298	1.695	1.968	.8830	.5042	.9958	15.13
.06000	.1043	.6553	.5753	.7033	1.2225	.8180	1.311	1.719	1.989	.8811	.5068	.9932	14.91
.06100	.1053	.6616	.5794	.7110	1.2270	.8150	1.3231	1.744	2.011	.8792	.5094	.9907	14.70
.06200	.1063	.6678	.5834	.7187	1.2315	.8121	1.336	1.770	2.033	.8773	.5119	.9883	14.50
.06300	.1073	.6739	.5874	.7256	1.2355	.8093	1.348	1.795	2.055	.8755	.5143	.9860	14.30
.06400	.1082	.6799	.5914	.7335	1.2402	.8063	1.360	1.819	2.076	.8737	.5167	.9837	14.11
.06500	.1092	.6860	.5954	.7411	1.2447	.8035	1.372	1.845	2.098	.8719	.5191	.9815	13.92
.06600	.1101	.6920	.5993	.7486	1.2492	.8005	1.384	1.870	2.121	.8700	.5214	.9793	13.74
.06700	.1111	.6981	.6031	.7561	1.2537	.7977	1.396	1.896	2.144	.8682	.5236	.9772	13.57
.06800	.1120	.7037	.6069	.7633	1.2580	.7948	1.408	1.921	2.166	.8664	.5258	.9752	13.40
.06900	.1130	.7099	.6106	.7711	1.2628	.7919	1.420	1.948	2.189	.8646	.5279	.9732	13.24
.07000	.1139	.7157	.6144	.7783	1.2672	.7890	1.432	1.974	2.213	.8627	.5300	.9713	13.08
.07100	.1149	.7219	.6181	.7863	1.2721	.7861	1.444	2.000	2.236	.8609	.5321	.9694	12.92
.07200	.1158	.7277	.6217	.7937	1.2767	.7833	1.455	2.026	2.260	.8591	.5341	.9676	12.77
.07300	.1168	.7336	.6252	.8011	1.2813	.7804	1.467	2.053	2.284	.8572	.5360	.9658	12.62
.07400	.1177	.7395	.6289	.8088	1.2861	.7775	1.479	2.080	2.308	.8554	.5380	.9641	12.48
.07500	.1186	.7453	.6324	.8162	1.2908	.7747	1.490	2.107	2.332	.8537	.5399	.9624	12.34
.07600	.1195	.7511	.6359	.8237	1.2956	.7719	1.502	2.135	2.357	.8519	.5417	.9607	12.21
.07700	.1205	.7569	.6392	.8312	1.3004	.7690	1.514	2.162	2.382	.8501	.5435	.9591	12.08
.07800	.1214	.7625	.6427	.8386	1.3051	.7662	1.525	2.189	2.407	.8483	.5452	.9576	11.95
.07900	.1223	.7683	.6460	.8462	1.3100	.7634	1.537	2.217	2.432	.8465	.5469	.9562	11.83
.08000	.1232	.7741	.6493	.8538	1.3149	.7605	1.548	2.245	2.458	.8448	.5485	.9548	11.71
.08100	.1241	.7799	.6526	.8614	1.3198	.7577	1.560	2.274	2.484	.8430	.5501	.9534	11.59
.08200	.1251	.7854	.6558	.8687	1.3246	.7549	1.571	2.303	2.511	.8413	.5517	.9520	11.47
.08300	.1259	.7911	.6590	.8762	1.3295	.7522	1.583	2.331	2.537	.8395	.5533	.9506	11.36
.08400	.1268	.7967	.6622	.8837	1.3345	.7494	1.594	2.360	2.563	.8378	.5548	.9493	11.25
.08500	.1277	.8026	.6655	.8915	1.3397	.7464	1.605	2.389	2.590	.8360	.5563	.9481	11.14
.08600	.1286	.8080	.6685	.8989	1.3446	.7437	1.616	2.418	2.617	.8342	.5577	.9469	11.04
.08700	.1295	.8137	.6716	.9064	1.3497	.7409	1.628	2.448	2.644	.8325	.5591	.9457	10.94
.08800	.1304	.8193	.6747	.9141	1.3548	.7381	1.639	2.478	2.672	.8308	.5605	.9445	10.84
.08900	.1313	.8250	.6778	.9218	1.3600	.7353	1.650	2.508	2.700	.8290	.5619	.9433	10.74
.09000	.1322	.8306	.6808	.9295	1.3653	.7324	1.661	2.538	2.728	.8273	.5632	.9422	10.65
.09100	.1331	.8363	.6838	.9372	1.3706	.7296	1.672	2.568	2.756	.8255	.5645	.9411	10.55
.09200	.1340	.8420	.6868	.9450	1.3759	.7268	1.684	2.599	2.785	.8238	.5658	.9401	10.46
.09300	.1349	.8474	.6897	.9525	1.3810	.7241	1.695	2.630	2.814	.8221	.5670	.9391	10.37
.09400	.1357	.8528	.6925	.9600	1.3862	.7214	1.706	2.662	2.843	.8204	.5682	.9381	10.29
.09500	.1366	.8583	.6953	.9677	1.3917	.7186	1.717	2.693	2.873	.8187	.5693	.9371	10.21
.09600	.1375	.8639	.6982	.9755	1.3970	.7158	1.728	2.726	2.903	.8170	.5704	.9362	10.12
.09700	.1384	.8694	.7011	.9832	1.4023	.7131	1.739	2.757	2.933	.8153	.5716	.9353	10.04
.09800	.1392	.8749	.7039	.9908	1.4077	.7104	1.750	2.790	2.963	.8136	.5727	.9344	9.962
.09900	.1401	.8803	.7066	.9985	1.4131	.7076	1.761	2.822	2.994	.8120	.5737	.9335	9.884
.1000	.1410	.8858	.7093	1.006	1.4187	.7049	1.772	2.855	3.025	.8103	.5747	.9327	9.808
.1010	.1419	.8913	.7120	1.014	1.4242	.7022	1.783	2.888	3.057	.8086	.5757	.9319	9.734
.1020	.1427	.8967	.7147	1.022	1.4297	.6994	1.793	2.922	3.088	.8069	.5766	.9311	9.661
.1030	.1436	.9023	.7173	1.030	1.4354	.6967	1.805	2.956	3.121	.8052	.5776	.9304	9.590
.1040	.1445	.9076	.7200	1.037	1.4410	.6940	1.815	2.990	3.153	.8036	.5785	.9297	9.519
.1050	.1453	.9130	.7226	1.045	1.4465	.6913	1.826	3.024	3.185	.8019	.5794	.9290	9.451
.1060	.1462	.9184	.7252	1.053	1.4523	.6886	1.837	3.059	3.218	.8003	.5803	.9282	9.384
.1070	.1470	.9239	.7277	1.061	1.4580	.6859	1.848	3.094	3.251	.7986	.5812	.9276	9.318
.1080	.1479	.9293	.7303	1.069	1.4638	.6833	1.858	3.128	3.284	.7970	.5820	.9269	9.254
.1090	.1488	.9343	.7327	1.076	1.4692	.6806	1.869	3.164	3.319	.7954	.5828	.9263	9.191
.1100	.1496	.9400	.7352	1.085	1.4752	.6779	1.880	3.201	3.353	.7937	.5836	.9257	9.129
.1110	.1505	.9456	.7377	1.093	1.4814	.6752	1.891	3.237	3.388	.7920	.5843	.9251	9.068
.1120	.1513	.9508	.7402	1.101	1.4871	.6725	1.902	3.274	3.423	.7904	.5850	.9245	9.009
.1130	.1522	.9563	.7426	1.109	1.4932	.6697	1.913	3.312	3.459	.7888	.5857	.9239	8.950
.1140	.1530	.9616	.7450	1.117	1.4990	.6671	1.923	3.348	3.494	.7872	.5864	.9234	8.891
.1150	.1539	.9670	.7474	1.125	1.5051	.6645	1.934	3.385	3.530	.7856	.5871	.9228	8.835
.1160	.1547	.9720	.7497	1.133	1.5108	.6619	1.944	3.423	3.566	.7840	.5878	.9223	8.780
.1170	.1556	.9775	.7520	1.141	1.5171	.6592	1.955	3.462	3.603	.7824	.5884	.9218	8.726
.1180	.1564	.9827	.7543	1.149	1.5230	.6566	1.966	3.501	3.641	.7808	.5890	.9214	8.673
.1190	.1573	.9882	.7566	1.157	1.5293	.6539	1.977	3.540	3.678	.7792	.5896	.9209	8.621

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H'_0	M
.1200	.1581	.9936	.7589	1.165	1.5356	.6512	1.987	3.579	3.716	.7776	.5902	.9204	8.569
.1210	.1590	.9989	.7612	1.174	1.5418	.6486	1.998	3.620	3.755	.7760	.5907	.9200	8.518
.1220	.1598	1.004	.7634	1.182	1.5479	.6460	2.008	3.659	3.793	.7745	.5913	.9196	8.468
.1230	.1607	1.010	.7656	1.190	1.5546	.6433	2.019	3.699	3.832	.7729	.5918	.9192	8.419
.1240	.1615	1.015	.7678	1.198	1.5605	.6407	2.030	3.740	3.871	.7713	.5922	.9189	8.371
.1250	.1624	1.020	.7700	1.207	1.5674	.6381	2.041	3.782	3.912	.7698	.5926	.9186	8.324
.1260	.1632	1.025	.7721	1.215	1.5734	.6356	2.051	3.824	3.952	.7682	.5931	.9182	8.278
.1270	.1640	1.030	.7742	1.223	1.5795	.6331	2.061	3.865	3.992	.7667	.5936	.9178	8.233
.1280	.1649	1.036	.7763	1.231	1.5862	.6305	2.072	3.907	4.033	.7652	.5940	.9175	8.189
.1290	.1657	1.041	.7783	1.240	1.5927	.6279	2.082	3.950	4.074	.7637	.5944	.9172	8.146
.1300	.1665	1.046	.7804	1.248	1.5990	.6254	2.093	3.992	4.115	.7621	.5948	.9169	8.103
.1310	.1674	1.052	.7824	1.257	1.6060	.6228	2.104	4.036	4.158	.7606	.5951	.9166	8.061
.1320	.1682	1.057	.7844	1.265	1.6124	.6202	2.114	4.080	4.201	.7591	.5954	.9164	8.020
.1330	.1691	1.062	.7865	1.273	1.6191	.6176	2.125	4.125	4.245	.7575	.5958	.9161	7.978
.1340	.1699	1.068	.7885	1.282	1.6260	.6150	2.135	4.169	4.288	.7560	.5961	.9158	7.937
.1350	.1708	1.073	.7905	1.291	1.633	.6123	2.146	4.217	4.334	.7545	.5964	.9156	7.897
.1360	.1716	1.078	.7925	1.300	1.640	.6098	2.156	4.262	4.378	.7530	.5967	.9154	7.857
.1370	.1724	1.084	.7945	1.308	1.647	.6073	2.167	4.309	4.423	.7515	.5969	.9152	7.819
.1380	.1733	1.089	.7964	1.317	1.654	.6047	2.177	4.355	4.468	.7500	.5972	.9150	7.781
.1390	.1741	1.094	.7983	1.326	1.660	.6022	2.188	4.402	4.514	.7485	.5975	.9148	7.744
.1400	.1749	1.099	.8002	1.334	1.667	.5998	2.198	4.450	4.561	.7471	.5978	.9146	7.707
.1410	.1758	1.105	.8021	1.343	1.675	.5972	2.209	4.498	4.607	.7456	.5980	.9144	7.671
.1420	.1766	1.110	.8039	1.352	1.681	.5947	2.219	4.546	4.654	.7441	.5982	.9142	7.636
.1430	.1774	1.115	.8057	1.360	1.688	.5923	2.230	4.595	4.663	.7426	.5984	.9141	7.602
.1440	.1783	1.120	.8076	1.369	1.696	.5898	2.240	4.644	4.751	.7412	.5986	.9140	7.567
.1450	.1791	1.125	.8094	1.378	1.703	.5873	2.251	4.695	4.800	.7397	.5987	.9139	7.533
.1460	.1800	1.131	.8112	1.388	1.710	.5847	2.261	4.746	4.850	.7382	.5989	.9137	7.499
.1470	.1808	1.136	.8131	1.397	1.718	.5822	2.272	4.798	4.901	.7368	.5990	.9136	7.465
.1480	.1816	1.141	.8149	1.405	1.725	.5798	2.282	4.847	4.951	.7354	.5992	.9135	7.432
.1490	.1825	1.146	.8166	1.415	1.732	.5773	2.293	4.901	5.001	.7339	.5993	.9134	7.400
.1500	.1833	1.152	.8183	1.424	1.740	.5748	2.303	4.954	5.054	.7325	.5994	.9133	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.5723	2.314	5.007	5.106	.7311	.5994	.9133	7.339
.1520	.1850	1.162	.8217	1.442	1.755	.5699	2.324	5.061	5.159	.7296	.5995	.9132	7.309
.1530	.1858	1.167	.8234	1.451	1.762	.5675	2.335	5.115	5.212	.7282	.5996	.9132	7.279
.1540	.1866	1.173	.8250	1.460	1.770	.5651	2.345	5.169	5.265	.7268	.5996	.9132	7.250
.1550	.1875	1.178	.8267	1.469	1.777	.5627	2.356	5.225	5.320	.7254	.5997	.9131	7.221
.1560	.1883	1.183	.8284	1.479	1.785	.5602	2.366	5.283	5.376	.7240	.5998	.9130	7.191
.1570	.1891	1.188	.8301	1.488	1.793	.5577	2.377	5.339	5.432	.7226	.5999	.9129	7.162
.1580	.1900	1.194	.8317	1.498	1.801	.5552	2.387	5.398	5.490	.7212	.5998	.9130	7.134
.1590	.1908	1.199	.8333	1.507	1.809	.5528	2.398	5.454	5.544	.7198	.5998	.9130	7.107
.1600	.1917	1.204	.8349	1.517	1.817	.5504	2.408	5.513	5.603	.7184	.5998	.9130	7.079
.1610	.1925	1.209	.8365	1.527	1.825	.5480	2.419	5.571	5.660	.7171	.5998	.9130	7.052
.1620	.1933	1.215	.8381	1.536	1.833	.5456	2.429	5.630	5.718	.7157	.5998	.9130	7.026
.1630	.1941	1.220	.8396	1.546	1.841	.5432	2.440	5.690	5.777	.7144	.5998	.9130	7.000
.1640	.1950	1.225	.8411	1.555	1.849	.5409	2.450	5.751	5.837	.7130	.5998	.9130	6.975
.1650	.1958	1.230	.8427	1.565	1.857	.5385	2.461	5.813	5.898	.7117	.5997	.9131	6.949
.1660	.1966	1.235	.8442	1.574	1.865	.5362	2.471	5.874	5.959	.7103	.5996	.9132	6.924
.1670	.1975	1.240	.8457	1.584	1.873	.5339	2.482	5.938	6.021	.7090	.5996	.9132	6.900
.1680	.1983	1.246	.8472	1.594	1.882	.5315	2.492	6.003	6.085	.7076	.5995	.9133	6.876
.1690	.1992	1.251	.8486	1.604	1.890	.5291	2.503	6.066	6.148	.7063	.5994	.9133	6.853
.1700	.2000	1.257	.8501	1.614	1.899	.5267	2.513	6.130	6.212	.7050	.5993	.9134	6.830
.1710	.2008	1.262	.8515	1.624	1.907	.5243	2.523	6.197	6.275	.7036	.5992	.9135	6.807
.1720	.2017	1.267	.8529	1.634	1.915	.5220	2.534	6.262	6.342	.7023	.5991	.9136	6.784
.1730	.2025	1.272	.8544	1.644	1.924	.5197	2.544	6.329	6.407	.7010	.5989	.9137	6.761
.1740	.2033	1.277	.8558	1.654	1.933	.5174	2.555	6.395	6.473	.6997	.5988	.9138	6.738
.1750	.2042	1.282	.8572	1.664	1.941	.5151	2.565	6.465	6.541	.6984	.5987	.9139	6.716
.1760	.2050	1.288	.8586	1.675	1.951	.5127	2.576	6.534	6.610	.6971	.5985	.9140	6.694
.1770	.2058	1.293	.8600	1.685	1.959	.5104	3.586	6.603	6.679	.6958	.5984	.9141	6.672
.1780	.2066	1.298	.8614	1.695	1.968	.5081	2.597	6.672	6.747	.6946	.5982	.9142	6.651
.1790	.2075	1.304	.8627	1.706	1.977	.5058	2.607	6.744	6.818	.6933	.5980	.9144	6.631
.1800	.2083	1.309	.8640	1.716	1.986	.5036	2.618	6.818	6.891	.6920	.5979	.9145	6.611
.1810	.2092	1.314	.8653	1.727	1.995	.5013	2.629	6.890	6.963	.6907	.5977	.9146	6.591
.1820	.2100	1.320	.8666	1.737	2.004	.4990	2.639	6.963	7.035	.6895	.5975	.9148	6.571
.1830	.2108	1.325	.8680	1.748	2.013	.4967	2.650	7.038	7.109	.6882	.5974	.9149	6.550
.1840	.2117	1.330	.8693	1.758	2.022	.4945	2.660	7.113	7.183	.6870	.5972	.9150	6.530
.1850	.2125	1.335	.8706	1.769	2.032	.4922	2.671	7.191	7.260	.6857	.5969	.9152	6.511
.1860	.2134	1.341	.8718	1.780	2.041	.4899	2.681	7.267	7.336	.6845	.5967	.9154	6.492
.1870	.2142	1.346	.8731	1.791	2.051	.4876	2.692	7.345	7.412	.6832	.5965	.9155	6.474
.1880	.2150	1.351	.8743	1.801	2.060	.4854	2.702	7.421	7.488	.6820	.5963	.9157	6.456
.1890	.2159	1.356	.8755	1.812	2.070	.4832	2.712	7.500	7.566	.6808	.5961	.9159	6.438

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.1900	.2167	1.362	.8767	1.823	2.079	.4809	2.723	7.581	7.647	.6796	.5958	.9161	6.421
.1910	.2176	1.367	.8779	1.834	2.089	.4787	2.734	7.663	7.728	.6784	.5955	.9163	6.403
.1920	.2184	1.372	.8791	1.845	2.099	.4765	2.744	7.746	7.810	.6772	.5952	.9165	6.385
.1930	.2192	1.377	.8803	1.856	2.108	.4743	2.755	7.827	7.891	.6760	.5950	.9167	6.368
.1940	.2201	1.383	.8815	1.867	2.118	.4721	2.765	7.911	7.974	.6748	.5948	.9169	6.351
.1950	.2209	1.388	.8827	1.879	2.128	.4699	2.776	7.996	8.059	.6736	.5946	.9170	6.334
.1960	.2218	1.393	.8839	1.890	2.138	.4677	2.787	8.083	8.145	.6724	.5944	.9172	6.317
.1970	.2226	1.399	.8850	1.901	2.148	.4655	2.797	8.167	8.228	.6712	.5941	.9174	6.300
.1980	.2234	1.404	.8862	1.913	2.158	.4633	2.808	8.256	8.316	.6700	.5938	.9176	6.284
.1990	.2243	1.409	.8873	1.924	2.169	.4611	2.819	8.346	8.406	.6689	.5935	.9179	6.268
.2000	.2251	1.414	.8884	1.935	2.178	.4590	2.829	8.436	8.495	.6677	.5932	.9181	6.253
.2010	.2260	1.420	.8895	1.947	2.189	.4569	2.840	8.524	8.583	.6666	.5929	.9183	6.237
.2020	.2268	1.425	.8906	1.959	2.199	.4547	2.850	8.616	8.674	.6654	.5926	.9186	6.222
.2030	.2277	1.430	.8917	1.970	2.210	.4526	2.861	8.708	8.766	.6642	.5923	.9188	6.206
.2040	.2285	1.436	.8928	1.982	2.220	.4504	2.872	8.803	8.860	.6631	.5920	.9190	6.191
.2050	.2293	1.441	.8939	1.994	2.231	.4483	2.882	8.897	8.953	.6620	.5917	.9193	6.176
.2060	.2302	1.446	.8950	2.006	2.242	.4462	2.893	8.994	9.050	.6608	.5914	.9195	6.161
.2070	.2310	1.451	.8960	2.017	2.252	.4441	2.903	9.090	9.144	.6597	.5911	.9197	6.147
.2080	.2319	1.457	.8971	2.030	2.263	.4419	2.914	9.187	9.240	.6586	.5908	.9200	6.133
.2090	.2328	1.462	.8981	2.042	2.274	.4398	2.925	9.288	9.342	.6574	.5905	.9202	6.119
.2100	.2336	1.468	.8991	2.055	2.285	.4377	2.936	9.389	9.442	.6563	.5901	.9205	6.105
.2110	.2344	1.473	.9001	2.066	2.295	.4357	2.946	9.490	9.542	.6552	.5898	.9207	6.091
.2120	.2353	1.479	.9011	2.079	2.307	.4336	2.957	9.590	9.642	.6541	.5894	.9210	6.077
.2130	.2361	1.484	.9021	2.091	2.318	.4315	2.967	9.693	9.744	.6531	.5891	.9213	6.064
.2140	.2370	1.489	.9031	2.103	2.329	.4294	2.978	9.796	9.847	.6520	.5888	.9215	6.051
.2150	.2378	1.494	.9041	2.115	2.340	.4274	2.989	9.902	9.952	.6509	.5884	.9218	6.037
.2160	.2387	1.500	.9051	2.128	2.351	.4253	2.999	10.01	10.06	.6498	.5881	.9221	6.024
.2170	.2395	1.506	.9061	2.142	2.364	.4232	3.010	10.12	10.17	.6488	.5878	.9223	6.011
.2180	.2404	1.511	.9070	2.154	2.375	.4211	3.021	10.23	10.28	.6477	.5874	.9226	5.999
.2190	.2412	1.516	.9079	2.166	2.386	.4191	3.031	10.34	10.38	.6467	.5871	.9228	5.987
.2200	.2421	1.521	.9088	2.178	2.397	.4171	3.042	10.45	10.50	.6456	.5868	.9231	5.975
.2210	.2429	1.526	.9097	2.192	2.409	.4151	3.052	10.56	10.61	.6446	.5864	.9234	5.963
.2220	.2438	1.532	.9107	2.204	2.421	.4131	3.063	10.68	10.72	.6436	.5861	.9236	5.951
.2230	.2446	1.537	.9116	2.218	2.433	.4111	3.074	10.79	10.84	.6425	.5857	.9239	5.939
.2240	.2455	1.542	.9125	2.230	2.444	.4091	3.085	10.91	10.95	.6414	.5854	.9242	5.927
.2250	.2463	1.548	.9134	1.244	2.457	.4071	3.095	11.02	11.07	.6404	.5850	.9245	5.915
.2260	.2472	1.553	.9143	2.257	2.469	.4051	3.106	11.15	11.19	.6394	.5846	.9248	5.903
.2270	.2481	1.559	.9152	2.271	2.481	.4031	3.117	11.27	11.31	.6383	.5842	.9251	5.891
.2280	.2489	1.564	.9161	2.284	2.493	.4011	3.128	11.39	11.44	.6373	.5838	.9254	5.880
.2290	.2498	1.569	.9170	2.297	2.506	.3991	3.138	11.51	11.56	.6363	.5834	.9258	5.869
.2300	.2506	1.575	.9178	2.311	2.518	.3971	3.149	11.64	11.68	.6353	.5830	.9261	5.858
.2310	.2515	1.580	.9186	2.325	2.531	.3952	3.160	11.77	11.81	.6343	.5826	.9264	5.848
.2320	.2523	1.585	.9194	2.338	2.543	.3932	3.171	11.90	11.93	.6333	.5823	.9267	5.838
.2330	.2532	1.591	.9203	2.352	2.556	.3912	3.182	12.03	12.07	.6323	.5819	.9270	5.827
.2340	.2540	1.596	.9211	2.366	2.569	.3893	3.192	12.15	12.19	.6313	.5815	.9273	5.816
.2350	.2549	1.602	.9219	2.380	2.581	.3874	3.203	12.29	12.33	.6304	.5811	.9276	5.806
.2360	.2558	1.607	.9227	2.393	2.594	.3855	3.214	12.43	12.47	.6294	.5807	.9279	5.796
.2370	.2566	1.612	.9235	2.408	2.607	.3836	3.225	12.55	12.59	.6284	.5804	.9282	5.786
.2380	.2575	1.618	.9243	2.422	2.620	.3816	3.236	12.69	12.73	.6275	.5800	.9285	5.776
.2390	.2584	1.623	.9251	2.436	2.634	.3797	3.247	12.83	12.87	.6265	.5796	.9288	5.766
.2400	.2592	1.629	.9259	2.450	2.647	.3779	3.257	12.97	13.01	.6256	.5792	.9291	5.756
.2410	.2601	1.634	.9267	2.464	2.660	.3760	3.268	13.11	13.15	.6246	.5788	.9294	5.746
.2420	.2610	1.640	.9275	2.480	2.674	.3741	3.279	13.26	13.30	.6237	.5784	.9298	5.736
.2430	.2618	1.645	.9282	2.494	2.687	.3722	3.290	13.40	13.44	.6228	.5780	.9301	5.727
.2440	.2627	1.650	.9289	2.508	2.700	.3704	3.301	13.55	13.59	.6218	.5776	.9304	5.718
.2450	.2635	1.656	.9296	2.523	2.714	.3685	3.312	13.70	13.73	.6209	.5772	.9307	5.710
.2460	.2644	1.661	.9304	2.538	2.728	.3666	3.323	13.85	13.88	.6200	.5768	.9310	5.701
.2470	.2653	1.667	.9311	2.553	2.742	.3648	3.334	14.00	14.04	.6191	.5764	.9314	5.692
.2480	.2661	1.672	.9318	2.568	2.755	.3629	3.344	14.15	14.19	.6182	.5760	.9317	5.684
.2490	.2670	1.678	.9325	2.583	2.770	.3610	3.355	14.31	14.35	.6173	.5756	.9320	5.675
.2500	.2679	1.683	.9332	2.599	2.784	.3592	3.367	14.47	14.51	.6164	.5752	.9323	5.667
.2510	.2687	1.689	.9339	2.614	2.798	.3574	3.377	14.62	14.66	.6155	.5748	.9327	5.658
.2520	.2696	1.694	.9346	2.629	2.813	.3556	3.388	14.79	14.82	.6146	.5744	.9330	5.650
.2530	.2705	1.700	.9353	2.645	2.828	.3537	3.399	14.95	14.99	.6137	.5740	.9333	5.641
.2540	.2714	1.705	.9360	2.660	2.842	.3519	3.410	15.12	15.15	.6128	.5736	.9336	5.633
.2550	.2722	1.711	.9367	2.676	2.856	.3501	3.421	15.29	15.32	.6120	.5732	.9340	5.624
.2560	.2731	1.716	.9374	2.691	2.871	.3483	3.432	15.45	15.49	.6111	.5728	.9343	5.616
.2570	.2740	1.722	.9381	2.707	2.886	.3465	3.443	15.63	15.66	.6102	.5724	.9346	5.608
.2580	.2749	1.727	.9388	2.723	2.901	.3447	3.454	15.80	15.83	.6093	.5720	.9349	5.600
.2590	.2757	1.732	.9394	2.739	2.916	.3430	3.465	15.97	16.00	.6085	.5716	.9353	5.592

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
.2600	.2766	1.738	.9400	2.755	2.931	.3412	3.476	16.15	16.18	.6076	.5712	.9356	5.585
.2610	.2775	1.744	.9406	2.772	2.946	.3394	3.487	16.33	16.36	.6068	.5707	.9360	5.578
.2620	.2784	1.749	.9412	2.788	2.962	.3376	3.498	16.51	16.54	.6060	.5703	.9363	5.571
.2630	.2792	1.755	.9418	2.804	2.977	.3359	3.509	16.69	16.73	.6052	.5699	.9367	5.563
.2640	.2801	1.760	.9425	2.820	2.992	.3342	3.520	16.88	16.91	.6043	.5695	.9370	5.556
.2650	.2810	1.766	.9431	2.837	3.008	.3325	3.531	17.07	17.10	.6035	.5691	.9373	5.548
.2660	.2819	1.771	.9437	2.853	3.023	.3308	3.542	17.26	17.28	.6027	.5687	.9377	5.541
.2670	.2827	1.776	.9443	2.870	3.039	.3291	3.553	17.45	17.45	.6018	.5683	.9380	5.534
.2680	.2836	1.782	.9449	2.886	3.055	.3274	3.564	17.64	17.67	.6010	.5679	.9383	5.527
.2690	.2845	1.788	.9455	2.904	3.071	.3256	3.575	17.84	17.87	.6002	.5675	.9386	5.520
.2700	.2854	1.793	.9461	2.921	3.088	.3239	3.587	18.04	18.07	.5994	.5671	.9390	5.513
.2710	.2863	1.799	.9467	2.938	3.104	.3222	3.598	18.24	18.27	.5986	.5667	.9393	5.506
.2720	.2872	1.804	.9473	2.956	3.120	.3205	3.610	18.46	18.49	.5978	.5663	.9396	5.499
.2730	.2880	1.810	.9478	2.973	3.136	.3189	3.620	18.65	18.67	.5971	.5659	.9400	5.493
.2740	.2889	1.815	.9484	2.990	3.153	.3172	3.631	18.86	18.89	.5963	.5655	.9403	5.486
.2750	.2898	1.821	.9490	3.008	3.170	.3155	3.642	19.07	19.10	.5955	.5651	.9406	5.480
.2760	.2907	1.826	.9495	3.025	3.186	.3139	3.653	19.28	19.30	.5947	.5647	.9410	5.474
.2770	.2916	1.832	.9500	3.043	3.203	.3122	3.664	19.49	19.51	.5940	.5643	.9413	5.468
.2780	.2924	1.837	.9505	3.061	3.220	.3106	3.675	19.71	19.74	.5932	.5639	.9416	5.462
.2790	.2933	1.843	.9511	3.079	3.237	.3089	3.686	19.93	19.96	.5925	.5635	.9420	5.456
.2800	.2942	1.849	.9516	3.097	3.254	.3073	3.697	20.16	20.18	.5917	.5631	.9423	5.450
.2810	.2951	1.854	.9521	3.115	3.272	.3057	3.709	20.39	20.41	.5910	.5627	.9426	5.444
.2820	.2960	1.860	.9526	3.133	3.289	.3040	3.720	20.62	20.64	.5902	.5623	.9430	5.438
.2830	.2969	1.866	.9532	3.152	3.307	.3024	3.731	20.85	20.87	.5895	.5619	.9433	5.432
.2840	.2978	1.871	.9537	3.171	3.325	.3008	3.742	21.09	21.11	.5887	.5615	.9436	5.426
.2850	.2987	1.877	.9542	3.190	3.343	.2992	3.754	21.33	21.35	.5880	.5611	.9440	5.420
.2860	.2996	1.882	.9547	3.209	3.361	.2976	3.765	21.57	21.59	.5873	.5607	.9443	5.414
.2870	.3005	1.888	.9552	3.228	3.379	.2959	3.776	21.82	21.84	.5866	.5603	.9446	5.409
.2880	.3014	1.893	.9557	3.246	3.396	.2944	3.787	22.05	22.07	.5859	.5600	.9449	5.403
.2890	.3022	1.899	.9562	3.264	3.414	.2929	3.798	22.30	22.32	.5852	.5596	.9452	5.397
.2900	.3031	1.905	.9567	3.284	3.433	.2913	3.809	22.54	22.57	.5845	.5592	.9456	5.392
.2910	.3040	1.910	.9572	3.303	3.451	.2898	3.821	22.81	22.83	.5838	.5588	.9459	5.386
.2920	.3049	1.916	.9577	3.323	3.471	.2882	3.832	23.07	23.09	.5831	.5584	.9463	5.380
.2930	.3058	1.922	.9581	3.343	3.490	.2866	3.843	23.33	23.35	.5824	.5580	.9466	5.375
.2940	.3067	1.927	.9585	3.362	3.508	.2851	3.855	23.60	23.62	.5817	.5576	.9469	5.371
.2950	.3076	1.933	.9590	3.382	3.527	.2835	3.866	23.86	23.88	.5810	.5572	.9473	5.366
.2960	.3085	1.938	.9594	3.402	3.546	.2820	3.877	24.12	24.15	.5804	.5568	.9476	5.361
.2970	.3094	1.944	.9599	3.422	3.565	.2805	3.888	24.40	24.42	.5797	.5564	.9480	5.356
.2980	.3103	1.950	.9603	3.442	3.585	.2790	3.900	24.68	24.70	.5790	.5560	.9483	5.351
.2990	.3112	1.955	.9607	3.462	3.604	.2275	3.911	24.96	24.98	.5784	.5556	.9486	5.547
.3000	.3121	1.961	.9611	3.483	3.624	.2760	3.922	25.24	25.26	.5777	.5552	.9490	5.342
.3010	.3130	1.967	.9616	3.503	3.643	.2745	3.933	25.53	25.55	.5771	.5549	.9493	5.337
.3020	.3139	1.972	.9620	3.524	3.663	.2730	3.945	25.82	25.83	.5764	.5545	.9496	5.332
.3030	.3148	1.978	.9624	3.545	3.683	.2715	3.956	26.12	26.14	.5758	.5541	.9499	5.328
.3040	.3157	1.984	.9629	3.566	3.703	.2700	3.968	26.42	26.44	.5751	.5538	.9502	5.323
.3050	.3166	1.989	.9633	3.587	3.724	.2685	3.979	26.72	26.74	.5745	.5534	.9505	5.318
.3060	.3175	1.995	.9637	3.609	3.745	.2670	3.990	27.02	27.04	.5739	.5530	.9509	5.314
.3070	.3184	2.001	.9641	3.630	3.765	.2656	4.002	27.33	27.35	.5732	.5527	.9512	5.309
.3080	.3193	2.007	.9645	3.651	3.786	.2641	4.013	27.65	27.66	.5726	.5523	.9515	5.305
.3090	.3202	2.012	.9649	3.673	3.806	.2627	4.024	27.96	27.98	.5720	.5519	.9518	5.300
.3100	.3211	2.018	.9653	3.694	3.827	.2613	4.036	28.28	28.30	.5714	.5515	.9522	5.296
.3110	.3220	2.023	.9656	3.716	3.848	.2599	4.047	28.60	28.62	.5708	.5511	.9525	5.292
.3120	.3230	2.029	.9660	3.738	3.870	.2584	4.058	28.93	28.95	.5701	.5507	.9528	5.288
.3130	.3239	2.035	.9664	3.760	3.891	.2570	4.070	29.27	29.28	.5695	.5504	.9531	5.284
.3140	.3248	2.041	.9668	3.782	3.912	.2556	4.081	29.60	29.62	.5689	.5500	.9535	5.280
.3150	.3257	2.046	.9672	3.805	3.934	.2542	4.093	29.94	29.96	.5683	.5497	.9538	5.276
.3160	.3266	2.052	.9676	3.828	3.956	.2528	4.104	30.29	30.31	.5678	.5494	.9541	5.272
.3170	.3275	2.058	.9679	3.851	3.978	.2514	4.116	30.64	30.65	.5672	.5490	.9544	5.268
.3180	.3284	2.063	.9682	3.873	4.000	.2500	4.127	30.99	31.00	.5666	.5486	.9547	5.264
.3190	.3294	2.069	.9686	3.896	4.022	.2486	4.139	31.35	31.37	.5660	.5483	.9550	5.260
.3200	.3302	2.075	.9690	3.919	4.045	.2472	4.150	31.71	31.72	.5655	.5479	.9553	5.256
.3210	.3311	2.081	.9693	3.943	4.068	.2459	4.161	32.07	32.08	.5649	.5476	.9556	5.252
.3220	.3321	2.086	.9696	3.966	4.090	.2445	4.173	32.44	32.46	.5643	.5472	.9559	5.249
.3230	.3330	2.092	.9700	3.990	4.114	.2431	4.185	32.83	32.84	.5637	.5468	.9562	5.245
.3240	.3339	2.098	.9703	4.014	4.136	.2418	4.196	33.20	33.22	.5632	.5465	.9565	5.241
.3250	.3349	2.104	.9707	4.038	4.160	.2404	4.208	33.60	33.61	.5627	.5462	.9568	5.237
.3260	.3357	2.110	.9710	4.061	4.183	.2391	4.219	33.97	33.99	.5621	.5458	.9571	5.234
.3270	.3367	2.115	.9713	4.085	4.206	.2378	4.231	34.37	34.38	.5616	.5455	.9574	5.231
.3280	.3376	2.121	.9717	4.110	4.230	.2364	4.242	34.77	34.79	.5610	.5451	.9577	5.227
.3290	.3385	2.127	.9720	4.135	4.254	.2351	4.254	35.18	35.19	.5605	.5448	.9580	5.223

APPENDIX 1. (Continued)

d/L_o	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.3300	.3394	2.133	.9723	4.159	4.277	.2338	4.265	35.58	35.59	.5599	.5444	.9583	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.2325	4.277	35.99	36.00	.5594	.5441	.9586	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.2312	4.288	36.42	36.43	.5589	.5438	.9589	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.2299	4.300	36.84	36.85	.5584	.5434	.9592	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.2286	4.311	37.25	37.27	.5578	.5431	.9595	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.2273	4.323	37.70	37.72	.5573	.5427	.9598	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.2260	4.335	38.14	38.15	.5568	.5424	.9601	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.2247	4.346	38.59	38.60	.5563	.5421	.9604	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.2235	4.358	39.02	39.04	.5558	.5417	.9607	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.2222	4.369	39.48	39.49	.5553	.5414	.9610	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.2210	4.381	39.95	39.96	.5548	.5411	.9613	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.2198	4.392	40.40	40.41	.5544	.5408	.9615	5.185
.3420	.3504	2.202	.9758	4.466	4.576	.2185	4.404	40.87	40.89	.5539	.5405	.9618	5.182
.3430	.3514	2.208	.9761	4.492	4.602	.2173	4.416	41.36	41.37	.5534	.5402	.9621	5.179
.3440	.3523	2.214	.9764	4.521	4.630	.2160	4.427	41.85	41.84	.5529	.5399	.9623	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.2148	4.439	42.33	42.34	.5524	.5396	.9626	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.2136	4.451	42.83	42.84	.5519	.5392	.9629	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.2124	4.462	43.34	43.35	.5515	.5389	.9632	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.2111	4.474	43.85	43.86	.5510	.5386	.9635	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.2099	4.486	44.37	44.40	.5505	.5383	.9638	5.162
.3500	.3579	2.249	.9780	4.685	4.791	.2087	4.498	44.89	44.80	.5501	.5380	.9640	5.159
.3510	.3588	2.255	.9782	4.713	4.818	.2076	4.509	45.42	45.43	.5496	.5377	.9643	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.2064	4.521	45.95	45.96	.5492	.5374	.9646	5.154
.3530	.3607	2.266	.9787	4.770	4.873	.2052	4.533	46.50	46.51	.5487	.5371	.9648	5.152
.3540	.3616	2.272	.9790	4.798	4.901	.2040	4.544	47.03	47.04	.5483	.5368	.9651	5.149
.3550	.3625	2.278	.9792	4.827	4.929	.2029	4.556	47.59	47.60	.5479	.5365	.9654	5.147
.3560	.3635	2.284	.9795	4.856	4.957	.2017	4.568	48.15	48.16	.5474	.5362	.9657	5.144
.3570	.3644	2.290	.9797	4.885	4.987	.2005	4.579	48.72	48.73	.5470	.5359	.9659	5.141
.3580	.3653	2.296	.9799	4.914	5.015	.1994	4.591	49.29	49.30	.5466	.5356	.9662	5.139
.3590	.3663	2.301	.9801	4.944	5.044	.1983	4.603	49.88	49.89	.5461	.5353	.9665	5.137
.3600	.3672	2.307	.9804	4.974	5.072	.1972	4.615	50.47	50.48	.5457	.5350	.9667	5.134
.3610	.3682	2.313	.9806	5.004	5.103	.1960	4.627	51.08	51.09	.5453	.5347	.9670	5.132
.3620	.3691	2.319	.9808	5.034	5.132	.1949	4.638	51.67	51.67	.5449	.5344	.9673	5.130
.3630	.3700	2.325	.9811	5.063	5.161	.1938	4.650	52.27	52.28	.5445	.5342	.9675	5.127
.3640	.3709	2.331	.9813	5.094	5.191	.1926	4.661	52.89	52.90	.5441	.5339	.9677	5.125
.3650	.3719	2.337	.9815	5.124	5.221	.1915	4.673	53.52	53.53	.5437	.5336	.9680	5.123
.3660	.3728	2.342	.9817	5.155	5.251	.1904	4.685	54.15	54.16	.5433	.5333	.9683	5.121
.3670	.3737	2.348	.9819	5.186	5.281	.1894	4.697	54.78	54.79	.5429	.5330	.9686	5.118
.3680	.3747	2.354	.9821	5.217	5.312	.1883	4.708	55.42	55.43	.5425	.5327	.9688	5.116
.3690	.3756	2.360	.9823	5.248	5.343	.1872	4.720	56.09	56.10	.5421	.5325	.9690	5.114
.3700	.3766	2.366	.9825	5.280	5.374	.1861	4.732	56.76	56.77	.5417	.5322	.9693	5.112
.3710	.3775	2.372	.9827	5.312	5.406	.1850	4.744	57.43	57.44	.5413	.5319	.9696	5.110
.3720	.3785	2.378	.9830	5.345	5.438	.1839	4.756	58.13	58.14	.5409	.5317	.9698	5.107
.3730	.3794	2.384	.9832	5.377	5.469	.1828	4.768	58.82	58.83	.5405	.5314	.9700	5.105
.3740	.3804	2.390	.9834	5.410	5.502	.1818	4.780	59.52	59.53	.5402	.5312	.9702	5.103
.3750	.3813	2.396	.9835	5.443	5.534	.1807	4.792	60.24	60.25	.5398	.5309	.9705	5.101
.3760	.3822	2.402	.9837	5.475	5.566	.1797	4.803	60.95	60.95	.5394	.5306	.9707	5.099
.3770	.3832	2.408	.9839	5.508	5.598	.1786	4.815	61.68	61.68	.5390	.5304	.9709	5.097
.3780	.3841	2.413	.9841	5.541	5.631	.1776	4.827	62.41	62.42	.5387	.5301	.9712	5.095
.3790	.3850	2.419	.9843	5.572	5.661	.1766	4.838	63.13	63.14	.5383	.5299	.9714	5.093
.3800	.3860	2.425	.9845	5.609	5.697	.1756	4.851	63.90	63.91	.5380	.5296	.9717	5.091
.3810	.3869	2.431	.9847	5.643	5.731	.1745	4.862	64.66	64.67	.5376	.5294	.9719	5.090
.3820	.3879	2.437	.9848	5.677	5.765	.1735	4.875	65.45	65.46	.5372	.5291	.9721	5.088
.3830	.3888	2.443	.9850	5.712	5.798	.1725	4.885	66.20	66.21	.5369	.5288	.9724	5.086
.3840	.3898	2.449	.9852	5.746	5.833	.1715	4.898	67.00	67.01	.5365	.5286	.9726	5.084
.3850	.3907	2.455	.9854	5.780	5.866	.1705	4.910	67.80	67.81	.5362	.5284	.9728	5.082
.3860	.3917	2.461	.9855	5.814	5.900	.1695	4.922	68.61	68.62	.5359	.5281	.9730	5.081
.3870	.3926	2.467	.9857	5.850	5.935	.1685	4.934	69.45	69.46	.5355	.5279	.9732	5.079
.3880	.3936	2.473	.9859	5.886	5.970	.1675	4.946	70.28	70.29	.5352	.5276	.9735	5.077
.3890	.3945	2.479	.9860	5.921	6.005	.1665	4.958	71.12	71.13	.5349	.5274	.9737	5.076
.3900	.3955	2.485	.9862	5.957	6.040	.1656	4.970	71.97	71.98	.5345	.5271	.9739	5.074
.3910	.3964	2.491	.9864	5.993	6.076	.1646	4.982	72.85	72.86	.5342	.5269	.9741	5.072
.3920	.3974	2.497	.9865	6.029	6.112	.1636	4.993	73.72	73.72	.5339	.5267	.9743	5.071
.3930	.3983	2.503	.9867	6.066	6.148	.1627	5.005	74.59	74.59	.5336	.5265	.9745	5.069
.3940	.3993	2.509	.9869	6.103	6.185	.1617	5.017	75.48	75.48	.5332	.5262	.9748	5.067
.3950	.4002	2.515	.9870	6.140	6.221	.1608	5.029	76.40	76.40	.5329	.5260	.9750	5.066
.3960	.4012	2.521	.9872	6.177	6.258	.1598	5.041	77.32	77.32	.5326	.5258	.9752	5.064
.3970	.4021	2.527	.9873	6.215	6.295	.1589	5.053	78.24	78.24	.5323	.5255	.9754	5.063
.3980	.4031	2.532	.9874	6.252	6.332	.1579	5.065	79.19	79.19	.5320	.5253	.9756	5.062
.3990	.4040	2.538	.9876	6.290	6.369	.1570	5.077	80.13	80.13	.5317	.5251	.9758	5.060

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
.4000	.4050	2.544	.9877	6.329	6.407	.1561	5.089	81.12	81.12	.5314	.5248	.9761	5.058
.4010	.4059	2.550	.9879	6.367	6.445	.1552	5.101	82.08	82.08	.5311	.5246	.9763	5.056
.4020	.4069	2.556	.9880	6.406	6.483	.1542	5.113	83.06	83.06	.5308	.5244	.9765	5.055
.4030	.4078	2.562	.9882	6.444	6.521	.1533	5.125	84.07	84.07	.5305	.5242	.9766	5.053
.4040	.4088	2.568	.9883	6.484	6.561	.1524	5.137	85.11	85.11	.5302	.5240	.9768	5.052
.4050	.4098	2.575	.9885	6.525	6.601	.1515	5.149	86.14	86.14	.5299	.5238	.9777	5.050
.4060	.4107	2.581	.9886	6.564	6.640	.1506	5.161	87.17	87.17	.5296	.5236	.9772	5.049
.4070	.4116	2.586	.9887	6.603	6.679	.1497	5.173	88.20	88.20	.5293	.5234	.9774	5.048
.4080	.4126	2.592	.9889	6.644	6.718	.1488	5.185	89.28	89.28	.5290	.5232	.9776	5.046
.4090	.4136	2.598	.9890	6.684	6.758	.1480	5.197	90.39	90.39	.5287	.5229	.9778	5.045
.4100	.4145	2.604	.9891	6.725	6.799	.1471	5.209	91.44	91.44	.5285	.5227	.9780	5.044
.4110	.4155	2.610	.9892	6.766	6.839	.1462	5.221	92.55	92.55	.5282	.5225	.9782	5.043
.4120	.4164	2.616	.9894	6.806	6.879	.1454	5.233	93.67	93.67	.5279	.5223	.9784	5.041
.4130	.4174	2.623	.9895	6.849	6.921	.1445	5.245	94.83	94.83	.5277	.5221	.9786	5.040
.4140	.4183	2.629	.9896	6.890	6.963	.1436	5.257	95.96	95.96	.5274	.5219	.9788	5.039
.4150	.4193	2.635	.9898	6.932	7.004	.1428	5.269	97.13	97.13	.5271	.5217	.9790	5.037
.4160	.4203	2.641	.9899	6.974	7.046	.1419	5.281	98.30	98.30	.5269	.5215	.9792	5.036
.4170	.4212	2.647	.9900	7.018	7.088	.1411	5.294	99.52	99.52	.5266	.5213	.9794	5.035
.4180	.4222	2.653	.9901	7.060	7.130	.1403	5.305	100.7	100.7	.5263	.5211	.9795	5.034
.4190	.4231	2.659	.9902	7.102	7.173	.1394	5.317	101.9	101.9	.5261	.5209	.9797	5.033
.4200	.4241	2.665	.9904	7.146	7.215	.1386	5.329	103.1	103.1	.5258	.5208	.9798	5.031
.4210	.4251	2.671	.9905	7.190	7.259	.1378	5.341	104.4	104.4	.5256	.5206	.9800	5.030
.4220	.4260	2.677	.9906	7.234	7.303	.1369	5.353	105.7	105.7	.5253	.5204	.9802	5.029
.4230	.4270	2.683	.9907	7.279	7.349	.1361	5.366	107.0	107.0	.5251	.5202	.9804	5.028
.4240	.4280	2.689	.9908	7.325	7.392	.1353	5.378	108.3	108.3	.5248	.5200	.9806	5.027
.4250	.4289	2.695	.9909	7.371	7.438	.1345	5.390	109.7	109.7	.5246	.5198	.9808	5.026
.4260	.4298	2.701	.9910	7.412	7.479	.1337	5.402	110.9	110.9	.5244	.5196	.9810	5.025
.4270	.4308	2.707	.9911	7.457	7.524	.1329	5.414	112.2	112.2	.5241	.5195	.9811	5.024
.4280	.4318	2.713	.9912	7.503	7.570	.1321	5.426	113.6	113.6	.5239	.5193	.9812	5.023
.4290	.4328	2.719	.9913	7.550	7.616	.1313	5.438	115.0	115.0	.5237	.5191	.9814	5.022
.4300	.4337	2.725	.9914	7.595	7.661	.1305	5.450	116.4	116.4	.5234	.5189	.9816	5.021
.4310	.4347	2.731	.9915	7.642	7.707	.1298	5.462	117.8	117.8	.5232	.5187	.9818	5.020
.4320	.4356	2.737	.9916	7.688	7.753	.1290	5.474	119.2	119.2	.5230	.5186	.9819	5.019
.4330	.4366	2.743	.9917	7.735	7.800	.1282	5.486	120.7	120.7	.5227	.5184	.9821	5.018
.4340	.4376	2.749	.9918	7.783	7.847	.1274	5.499	122.2	122.2	.5225	.5182	.9823	5.017
.4350	.4385	2.755	.9919	7.831	7.895	.1267	5.511	123.7	123.7	.5223	.5181	.9824	5.016
.4360	.4395	2.762	.9920	7.880	7.943	.1259	5.523	125.2	125.2	.5221	.5179	.9826	5.015
.4370	.4405	2.768	.9921	7.922	7.991	.1251	5.535	126.7	126.7	.5218	.5177	.9828	5.014
.4380	.4414	2.774	.9922	7.975	8.035	.1244	5.547	128.3	128.3	.5216	.5176	.9829	5.013
.4390	.4424	2.780	.9923	8.026	8.088	.1236	5.560	129.9	129.9	.5214	.5174	.9830	5.012
.4400	.4434	2.786	.9924	8.075	8.136	.1229	5.572	131.4	131.4	.5212	.5172	.9832	5.011
.4410	.4443	2.792	.9925	8.124	8.185	.1222	5.584	133.0	133.0	.5210	.5171	.9833	5.010
.4420	.4453	2.798	.9926	8.175	8.236	.1214	5.596	134.7	134.7	.5208	.5169	.9835	5.009
.4430	.4463	2.804	.9927	8.228	8.285	.1207	5.608	136.3	136.3	.5206	.5168	.9836	5.008
.4440	.4472	2.810	.9928	8.274	8.334	.1200	5.620	137.9	137.9	.5204	.5166	.9838	5.007
.4450	.4482	2.816	.9929	8.326	8.387	.1192	5.632	139.6	139.6	.5202	.5165	.9839	5.006
.4460	.4492	2.822	.9930	8.379	8.438	.1185	5.644	141.4	141.4	.5200	.5163	.9841	5.005
.4470	.4501	2.828	.9930	8.427	8.486	.1178	5.657	143.1	143.1	.5198	.5161	.9843	5.005
.4480	.4511	2.834	.9931	8.481	8.540	.1171	5.669	144.8	144.8	.5196	.5160	.9844	5.004
.4490	.4521	2.840	.9932	8.532	8.590	.1164	5.681	146.6	146.6	.5194	.5158	.9846	5.003
.4500	.4531	2.847	.9933	8.585	8.643	.1157	5.693	148.4	148.4	.5192	.5157	.9847	5.002
.4510	.4540	2.853	.9934	8.638	8.695	.1150	5.705	150.2	150.2	.5190	.5156	.9848	5.001
.4520	.4550	2.859	.9935	8.693	8.750	.1143	5.717	152.1	152.1	.5188	.5154	.9849	5.000
.4530	.4560	2.865	.9935	8.747	8.804	.1136	5.730	154.0	154.0	.5186	.5152	.9851	5.000
.4540	.4569	2.871	.9936	8.797	8.854	.1129	5.742	155.9	155.9	.5184	.5151	.9852	4.999
.4550	.4579	2.877	.9937	8.853	8.910	.1122	5.754	157.7	157.7	.5182	.5150	.9853	4.998
.4560	.4589	2.883	.9938	8.910	8.965	.1115	5.766	159.7	159.7	.5181	.5148	.9855	4.997
.4570	.4599	2.890	.9938	8.965	9.021	.1109	5.779	161.7	161.7	.5179	.5146	.9857	4.997
.4580	.4608	2.896	.9939	9.016	9.072	.1102	5.791	163.6	163.6	.5177	.5145	.9858	4.996
.4590	.4618	2.902	.9940	9.074	9.129	.1095	5.803	165.6	165.6	.5175	.5144	.9859	4.995
.4600	.4628	2.908	.9941	9.132	9.186	.1089	5.815	167.7	167.7	.5173	.5143	.9860	4.994
.4610	.4637	2.914	.9941	9.183	9.238	.1083	5.827	169.7	169.7	.5172	.5141	.9862	4.994
.4620	.4647	2.920	.9942	9.242	9.296	.1076	5.840	171.8	171.8	.5170	.5140	.9863	4.993
.4630	.4657	2.926	.9943	9.301	9.354	.1069	5.852	173.9	173.9	.5168	.5139	.9864	4.992
.4640	.4666	2.932	.9944	9.353	9.406	.1063	5.864	176.0	176.0	.5167	.5138	.9865	4.991
.4650	.4676	2.938	.9944	9.413	9.466	.1056	5.876	178.2	178.2	.5165	.5136	.9867	4.991
.4660	.4686	2.944	.9945	9.472	9.525	.1050	5.888	180.4	180.4	.5163	.5135	.9868	4.990
.4670	.4695	2.951	.9946	9.533	9.585	.1043	5.900	182.6	182.6	.5162	.5134	.9869	4.989
.4680	.4705	2.957	.9946	9.586	9.638	.1037	5.912	184.8	184.8	.5160	.5132	.9871	4.989
.4690	.4715	2.963	.9947	9.647	9.699	.1031	5.925	187.2	187.2	.5158	.5131	.9872	4.988

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
.4700	.4725	2.969	.9947	9.709	9.760	.1025	5.937	189.5	189.5	.5157	.5129	.9873	4.988
.4710	.4735	2.975	.9948	9.770	9.821	.1018	5.949	191.8	191.8	.5155	.5128	.9874	4.987
.4720	.4744	2.981	.9949	9.826	9.877	.1012	5.962	194.2	194.2	.5154	.5127	.9875	4.986
.4730	.4754	2.987	.9949	9.888	9.938	.1006	5.974	196.5	196.5	.5152	.5126	.9876	4.986
.4740	.4764	2.993	.9950	9.951	10.00	.1000	5.986	199.0	199.0	.5150	.5125	.9877	4.985
.4750	.4774	2.999	.9951	10.01	10.07	.09942	5.999	201.4	201.4	.5149	.5124	.9878	4.984
.4760	.4783	3.005	.9951	10.07	10.12	.09882	6.011	203.9	203.9	.5147	.5122	.9880	4.984
.4770	.4793	3.012	.9952	10.13	10.18	.09820	6.023	206.5	206.5	.5146	.5121	.9881	4.983
.4780	.4803	3.018	.9952	10.20	10.25	.09759	6.036	209.0	209.0	.5144	.5120	.9882	4.983
.4790	.4813	3.024	.9953	10.26	10.31	.09698	6.048	211.7	211.7	.5143	.5119	.9883	4.982
.4800	.4822	3.030	.9953	10.32	10.37	.09641	6.060	214.2	214.2	.5142	.5117	.9885	4.982
.4810	.4832	3.036	.9954	10.39	10.43	.09583	6.072	216.8	216.8	.5140	.5116	.9886	4.981
.4820	.4842	3.042	.9955	10.45	10.50	.09523	6.085	219.5	219.5	.5139	.5115	.9887	4.980
.4830	.4852	3.049	.9955	10.52	10.57	.09464	6.097	222.2	222.2	.5137	.5114	.9888	4.980
.4840	.4862	3.055	.9956	10.59	10.63	.09405	6.109	225.0	225.0	.5136	.5113	.9889	4.979
.4850	.4871	3.061	.9956	10.65	10.69	.09352	6.121	228.3	228.3	.5134	.5112	.9890	4.979
.4860	.4881	3.067	.9957	10.71	10.76	.09294	6.134	230.6	230.6	.5133	.5111	.9891	4.978
.4870	.4891	3.073	.9957	10.78	10.83	.09236	6.146	233.5	233.5	.5132	.5110	.9892	4.978
.4880	.4901	3.079	.9958	10.85	10.90	.09178	6.159	236.4	236.4	.5130	.5109	.9893	4.977
.4890	.4911	3.086	.9958	10.92	10.96	.09121	6.171	239.6	239.6	.5129	.5107	.9895	4.977
.4900	.4920	3.092	.9959	10.99	11.03	.09064	6.183	242.3	242.3	.5128	.5106	.9896	4.976
.4910	.4930	3.098	.9959	11.05	11.09	.09010	6.195	245.2	245.2	.5126	.5105	.9897	4.976
.4920	.4940	3.104	.9960	11.12	11.16	.08956	6.208	248.3	248.3	.5125	.5104	.9898	4.975
.4930	.4950	3.110	.9960	11.19	11.24	.08901	6.220	251.3	251.3	.5124	.5103	.9899	4.975
.4940	.4960	3.117	.9961	11.26	11.31	.08845	6.232	254.5	254.5	.5122	.5102	.9899	4.974
.4950	.4969	3.122	.9961	11.32	11.37	.08793	6.245	257.6	257.6	.5121	.5101	.9900	4.974
.4960	.4979	3.128	.9962	11.40	11.44	.08741	6.257	260.8	260.8	.5120	.5100	.9901	4.973
.4970	.4989	3.135	.9962	11.47	11.51	.08691	6.269	264.0	264.0	.5119	.5099	.9902	4.973
.4980	.4999	3.141	.9963	11.54	11.59	.08637	6.282	267.3	267.3	.5118	.5098	.9903	4.972
.4990	.5009	3.147	.9963	11.61	11.65	.08584	6.294	270.6	270.6	.5116	.5097	.9904	4.972
.5000	.5018	3.153	.9964	11.68	11.72	.08530	6.306	274.0	274.0	.5115	.5096	.9905	4.971
.5010	.5028	3.159	.9964	11.75	11.80	.08477	6.319	277.5	277.5	.5114	.5095	.9906	4.971
.5020	.5038	3.166	.9964	11.83	11.87	.08424	6.331	280.8	280.8	.5113	.5094	.9907	4.971
.5030	.5048	3.172	.9965	11.91	11.95	.08371	6.343	284.3	284.3	.5112	.5093	.9908	4.970
.5040	.5058	3.178	.9965	11.98	12.02	.08320	6.356	287.9	287.9	.5110	.5092	.9909	4.970
.5050	.5067	3.184	.9966	12.05	12.09	.08270	6.368	291.4	291.4	.5109	.5092	.9909	4.969
.5060	.5077	3.190	.9966	12.12	12.16	.08220	6.380	295.0	295.0	.5108	.5091	.9910	4.969
.5070	.5087	3.196	.9967	12.20	12.24	.08169	6.393	298.7	298.7	.5107	.5090	.9911	4.968
.5080	.5097	3.203	.9967	12.28	12.32	.08119	6.405	302.4	302.4	.5106	.5089	.9912	4.968
.5090	.5107	3.209	.9968	12.35	12.39	.08068	6.417	306.2	306.2	.5105	.5088	.9913	4.967
.5100	.5117	3.215	.9968	12.43	12.47	.08022	6.430	310.0	310.0	.5104	.5087	.9914	4.967
.5110	.5126	3.221	.9968	12.50	12.54	.07972	6.442	313.8	313.8	.5103	.5086	.9915	4.967
.5120	.5136	3.227	.9969	12.58	12.62	.07922	6.454	317.7	317.7	.5102	.5086	.9915	4.966
.5130	.5146	3.233	.9969	12.66	12.70	.07873	6.467	321.7	321.7	.5101	.5085	.9916	4.966
.5140	.5156	3.240	.9970	12.74	12.78	.07824	6.479	325.7	325.7	.5100	.5084	.9917	4.965
.5150	.5166	3.246	.9970	12.82	12.86	.07776	6.491	329.7	329.7	.5098	.5083	.9918	4.965
.5160	.5176	3.252	.9970	12.90	12.94	.07729	6.504	333.8	333.8	.5097	.5082	.9919	4.965
.5170	.5185	3.258	.9971	12.98	13.02	.07682	6.516	337.9	337.9	.5096	.5082	.9919	4.964
.5180	.5195	3.264	.9971	13.06	13.10	.07634	6.529	342.2	342.2	.5095	.5081	.9920	4.964
.5190	.5205	3.270	.9971	13.14	13.18	.07587	6.541	346.4	346.4	.5094	.5080	.9921	4.964
.5200	.5215	3.277	.9972	13.22	13.26	.07540	6.553	350.7	350.7	.5093	.5079	.9922	4.963
.5210	.5225	3.283	.9972	13.31	13.35	.07494	6.566	355.1	355.1	.5092	.5078	.9923	4.963
.5220	.5235	3.289	.9972	13.39	13.43	.07449	6.578	359.6	359.6	.5092	.5077	.9924	4.963
.5230	.5244	3.295	.9973	13.47	13.51	.07404	6.590	364.0	364.0	.5091	.5077	.9924	4.962
.5240	.5254	3.301	.9973	13.55	13.59	.07358	6.603	368.5	368.5	.5090	.5076	.9925	4.962
.5250	.5264	3.308	.9973	13.64	13.68	.07312	6.615	373.1	373.1	.5089	.5075	.9926	4.962
.5260	.5274	3.314	.9974	13.73	13.76	.07266	6.628	377.8	377.8	.5088	.5074	.9927	4.961
.5270	.5284	3.320	.9974	13.81	13.85	.07221	6.640	382.5	382.5	.5087	.5074	.9927	4.961
.5280	.5294	3.326	.9974	13.90	13.94	.07177	6.652	387.3	387.3	.5086	.5073	.9928	4.961
.5290	.5304	3.333	.9975	13.99	14.02	.07134	6.665	392.2	392.2	.5085	.5072	.9929	4.960
.5300	.5314	3.339	.9975	14.07	14.10	.07091	6.677	397.0	397.0	.5084	.5071	.9930	4.960
.5310	.5323	3.345	.9975	14.16	14.19	.07047	6.690	402.0	402.0	.5083	.5070	.9931	4.960
.5320	.5333	3.351	.9976	14.25	14.28	.07003	6.702	406.9	406.9	.5082	.5070	.9931	4.959
.5330	.5343	3.357	.9976	14.34	14.37	.06959	6.714	412.0	412.0	.5082	.5069	.9932	4.959
.5340	.5353	3.363	.9976	14.43	14.46	.06915	6.727	417.2	417.2	.5081	.5068	.9933	4.959
.5350	.5363	3.370	.9976	14.52	14.55	.06872	7.639	422.4	422.4	.5080	.5068	.9933	4.959
.5360	.5373	3.376	.9977	14.61	14.64	.06829	6.752	427.7	427.7	.5079	.5067	.9934	4.958
.5370	.5383	3.382	.9977	14.70	14.73	.06787	6.764	433.1	433.1	.5078	.5066	.9935	4.958
.5380	.5393	3.388	.9977	14.79	14.82	.06746	6.776	438.5	438.5	.5077	.5066	.9935	4.958
.5390	.5402	3.394	.9977	14.88	14.91	.06705	6.789	444.0	444.0	.5077	.5065	.9936	4.958

APPENDIX 1. (Continued)

d/L_o	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.5400	.5412	3.401	.9978	14.97	15.01	.06664	6.801	449.5	449.5	.5076	.5065	.9936	4.957
.5410	.5422	3.407	.9978	15.07	15.10	.06623	6.814	455.1	455.1	.5075	.5064	.9937	4.957
.5420	.5432	3.413	.9978	15.16	15.19	.06582	6.826	460.7	460.7	.5074	.5063	.9938	4.957
.5430	.5442	3.419	.9979	15.25	15.29	.06542	6.838	466.4	466.4	.5073	.5063	.9938	4.956
.5440	.5452	3.426	.9979	15.35	15.38	.06501	6.851	472.2	472.2	.5073	.5062	.9939	4.956
.5450	.5461	3.432	.9979	15.45	15.48	.06461	6.863	478.1	478.1	.5072	.5061	.9940	4.956
.5460	.5471	3.438	.9979	15.54	15.58	.06420	6.876	484.3	484.3	.5071	.5060	.9941	4.956
.5470	.5481	3.444	.9980	15.64	15.67	.06380	6.888	490.3	490.3	.5070	.5060	.9941	4.955
.5480	.5491	3.450	.9980	15.74	15.77	.06341	6.901	496.4	496.4	.5070	.5059	.9942	4.955
.5490	.5501	3.456	.9980	15.84	15.87	.06302	6.913	502.5	502.5	.5069	.5059	.9942	4.955
.5500	.5511	3.463	.9980	15.94	15.97	.06263	6.925	508.7	508.7	.5068	.5058	.9942	4.955
.5510	.5521	3.469	.9981	16.04	16.07	.06224	6.937	515.0	515.0	.5067	.5058	.9942	4.954
.5520	.5531	3.475	.9981	16.14	16.17	.06186	6.950	521.6	521.6	.5067	.5057	.9943	4.954
.5530	.5541	3.481	.9981	16.24	16.27	.06148	6.962	528.1	528.1	.5066	.5056	.9944	4.954
.5540	.5551	3.488	.9981	16.34	16.37	.06110	6.975	534.8	534.8	.6065	.5056	.9944	4.954
.5550	.5560	3.494	.9982	16.44	16.47	.06073	6.987	541.4	541.4	.5065	.5056	.9945	4.953
.5560	.5570	3.500	.9982	16.54	16.57	.06035	7.000	548.1	548.1	.5064	.5055	.9945	4.653
.5570	.5580	3.506	.9982	16.65	16.68	.06997	7.012	554.9	554.9	.5063	.5054	.9946	4.953
.5580	.5590	3.512	.9982	16.75	16.78	.05960	7.025	562.0	562.0	.5063	.5053	.9947	4.953
.5590	.5600	3.519	.9982	16.85	16.88	.05923	7.037	569.1	569.1	.5062	.5053	.9947	4.953
.5600	.5610	3.525	.9983	16.96	16.99	.05887	7.050	576.1	576.1	.5061	.5053	.9947	4.952
.5610	.5620	3.531	.9983	17.06	17.09	.05850	7.062	583.3	583.3	.5061	.5052	.9948	4.952
.5620	.5630	3.537	.9983	17.17	17.20	.05814	7.074	590.7	590.7	.5060	.5051	.9949	4.952
.5630	.5640	3.543	.9983	17.28	17.31	.05778	7.087	598.0	598.0	.5059	.5051	.9949	4.952
.5640	.5649	3.550	.9984	17.38	17.41	.05743	7.099	605.0	605.0	.5059	.5050	.9950	4.951
.5650	.5659	3.556	.9984	17.49	17.52	.05707	7.112	613.2	613.2	.5058	.5050	.9950	4.951
.5660	.5669	3.562	.9984	17.60	17.63	.05672	7.124	620.8	620.8	.5057	.5049	.9951	4.951
.5670	.5679	3.568	.9984	17.71	17.74	.05637	7.136	628.5	628.5	.5057	.5049	.9951	4.951
.5680	.5689	3.575	.9984	17.82	17.85	.05602	7.149	636.4	636.4	.5056	.5048	.9952	4.951
.5690	.5699	3.581	.9985	17.94	17.97	.05567	7.161	644.3	644.3	.5056	.5048	.9952	4.950
.5700	.5709	3.587	.9985	18.05	18.08	.05532	7.174	652.4	652.4	.5055	.5047	.9953	4.950
.5710	.5719	3.593	.9985	18.16	18.19	.05497	7.186	660.5	660.5	.5054	.5047	.9953	4.950
.5720	.5729	3.600	.9985	18.28	18.31	.05463	7.199	668.8	668.8	.5054	.5046	.9954	4.950
.5730	.5738	3.606	.9985	18.39	18.42	.05430	7.211	677.2	677.2	.5053	.5046	.9954	4.950
.5740	.5748	3.612	.9985	18.50	18.53	.05396	7.224	685.6	685.6	.5053	.5045	.9955	4.950
.5750	.5758	3.618	.9986	18.62	18.64	.05363	7.236	694.3	694.3	.5052	.5045	.9955	4.949
.5760	.5768	3.624	.9986	18.73	18.76	.05330	7.249	703.2	703.2	.5052	.5044	.9956	4.949
.5770	.5778	3.630	.9986	18.85	18.88	.05297	7.261	711.9	711.9	.5051	.5044	.9956	4.949
.5780	.5788	3.637	.9986	18.97	19.00	.05264	7.274	720.8	720.8	.5051	.5043	.9957	4.949
.5790	.5798	3.643	.9986	19.09	19.12	.05231	7.286	729.9	729.9	.5050	.5043	.9957	4.949
.5800	.5808	3.649	.9987	19.21	19.24	.05198	7.298	739.0	739.0	.5049	.5043	.9957	4.948
.5810	.5818	3.656	.9987	19.33	19.36	.05166	7.311	748.1	748.1	.5049	.5042	.9955	4.948
.5820	.5828	3.662	.9987	19.45	19.48	.05134	7.323	757.5	757.5	.5048	.5042	.9958	4.948
.5830	.5838	3.668	.9987	19.58	19.60	.05102	7.336	767.0	767.0	.5048	.5041	.9959	4.948
.5840	.5848	3.674	.9987	19.70	19.73	.05070	7.348	776.7	776.7	.5047	.5041	.9959	4.948
.5850	.5858	3.680	.9987	19.81	19.84	.05040	7.361	786.5	786.5	.5047	.5040	.9960	4.948
.5860	.5867	3.686	.9987	19.94	19.96	.05009	7.373	796.4	796.4	.5046	.5040	.9960	4.948
.5870	.5877	3.693	.9988	20.06	20.09	.04978	7.386	806.5	806.5	.5046	.5040	.9960	4.947
.5880	.5887	3.699	.9988	20.19	20.21	.04947	7.398	816.5	816.5	.5045	.5039	.9961	4.947
.5890	.5897	3.705	.9988	20.32	20.34	.04916	7.411	826.7	826.7	.5045	.5039	.9961	4.947
.5900	.5907	3.712	.9988	20.45	20.47	.04885	7.423	837.1	837.1	.5044	.5038	.9962	4.947
.5910	.5917	3.718	.9988	20.57	20.60	.04855	7.436	847.6	847.6	.5044	.5038	.9962	4.947
.5920	.5927	3.724	.9988	20.70	20.73	.04824	7.448	858.2	858.2	.5043	.5037	.9963	4.947
.5930	.5937	3.730	.9989	20.83	20.86	.04794	7.460	868.9	868.9	.5043	.5037	.9963	4.946
.5940	.5947	3.737	.9989	20.97	20.99	.04764	7.473	879.8	879.8	.5043	.5037	.9963	4.946
.5950	.5957	3.743	.9989	21.10	21.12	.04735	7.485	890.8	890.8	.5042	.5036	.9964	4.946
.5960	.5967	3.749	.9989	21.23	21.25	.04706	7.498	901.9	901.9	.5042	.5036	.9964	4.946
.5970	.5977	3.755	.9989	21.35	21.37	.04677	7.510	913.4	913.4	.5041	.5036	.9964	4.946
.5980	.5987	3.761	.9989	21.49	21.51	.04648	7.523	925.0	925.0	.5041	.5035	.9965	4.946
.5990	.5996	3.767	.9989	21.62	21.64	.04619	7.535	936.5	936.5	.5040	.5035	.9965	4.946
.6000	.6006	3.774	.9990	21.76	21.78	.04591	7.548	948.1	948.1	.5040	.5035	.9965	4.945
.6100	.6106	3.836	.9991	23.17	23.19	.04313	7.673	1,074	1,074	.5036	.5031	.9969	4.944
.6200	.6205	3.899	.9992	24.66	24.68	.04052	7.798	1,217	1,217	.5032	.5028	.9972	4.943
.6300	.6305	3.961	.9993	26.25	26.27	.03806	7.923	1,379	1,379	.5029	.5025	.9975	4.942
.6400	.6404	4.024	.9994	27.95	27.97	.03576	8.048	1,527	1,527	.5026	.5023	.9977	4.941
.6500	.6504	4.086	.9994	29.75	29.77	.03359	8.173	1,771	1,771	.5023	.5020	.9980	4.940
.6600	.6603	4.149	.9995	31.68	31.69	.03155	8.298	2,008	2,008	.5021	.5018	.9982	4.940
.6700	.6703	4.212	.9996	33.73	33.74	.02964	8.423	2,275	2,275	.5019	.5017	.9983	4.939
.6800	.6803	4.274	.9996	35.90	35.92	.02784	8.548	2,579	2,579	.5017	.5015	.9985	4.939
.6900	.6902	4.337	.9997	38.23	38.24	.02615	8.674	2,923	2,923	.5015	.5013	.9987	4.938

APPENDIX 1. (Continued)

d/L_0	d/L	$2\pi d/L$	tanh $2\pi d/L$	sinh $2\pi d/L$	cosh $2\pi d/L$	K	$4\pi d/L$	sinh $4\pi d/L$	cosh $4\pi d/L$	n	C_G/C_0	H/H_0'	M
.7000	.7002	4.400	.9997	40.71	40.72	.02456	8.799	3,314	3,314	.5013	.5012	.9988	4.938
.7100	.7102	4.462	.9997	43.34	43.35	.02307	8.925	3,757	3,757	.5012	.5011	.9989	4.937
.7200	.7202	4.525	.9998	46.14	46.15	.02167	9.050	4,258	4,258	.5011	.5010	.9990	4.937
.7300	.7302	4.588	.9998	49.13	49.14	.02035	9.175	4,828	4,828	.5010	.5009	.9991	4.937
.7400	.7401	4.650	.9998	52.31	52.32	.01911	9.301	5,473	5,473	.5009	.5008	.9992	4.937
.7500	.7501	4.713	.9998	55.70	55.71	.01795	9.426	6,204	6,204	.5008	.5007	.9993	4.936
.7600	.7601	4.776	.9999	59.30	59.31	.01686	9.552	7,034	7,034	.5007	.5006	.9994	4.936
.7700	.7701	4.839	.9999	63.15	63.16	.01583	9.677	7,976	7,976	.5006	.5005	.9995	4.936
.7800	.7801	4.902	.9999	67.24	67.25	.01487	9.803	9,042	9,042	.5005	.5004	.9996	4.936
.7900	.7901	4.964	.9999	71.60	71.60	.01397	9.929	10,250	10,250	.5005	.5004	.9996	4.936
.8000	.8001	5.027	.9999	76.24	76.24	.01312	10.05	11,620	11,620	.5004	.5004	.9996	4.936
.8100	.8101	5.090	.9999	81.19	81.19	.01232	10.18	13,180	13,180	.5004	.5004	.9996	4.936
.8200	.8201	5.153	.9999	86.44	86.44	.01157	10.31	14,940	14,940	.5003	.5003	.9997	4.935
.8300	.8301	5.215	.9999	92.05	92.05	.01086	10.43	17,340	17,340	.5003	.5003	.9997	4.935
.8400	.8400	5.278	1.000	98.01	98.01	.01020	10.56	19,210	19,210	.5003	.5003	.9997	4.935
.8500	.8500	5.341	1.000	104.4	104.4	.009582	10.68	21,780	21,780	.5002	.5002	.9998	4.935
.8600	.8600	5.404	1.000	111.1	111.1	.009000	10.81	24,690	24,690	.5002	.5002	.9998	4.935
.8700	.8700	5.467	1.000	118.3	118.3	.008451	10.93	28,000	28,000	.5002	.5002	.9998	4.935
.8800	.8800	5.529	1.000	126.0	126.0	.007934	11.06	31,750	31,750	.5002	.5002	.9998	4.935
.8900	.8900	5.592	1.000	134.2	134.2	.007454	11.18	36,000	36,000	.5002	.5002	.9998	4.935
.9000	.9000	5.655	1.000	142.9	142.9	.007000	11.31	40,810	40,810	.5001	.5001	.9999	4.935
.9100	.9100	5.718	1.000	152.1	152.1	.006574	11.44	46,280	46,280	.5001	.5001	.9999	4.935
.9200	.9200	5.781	1.000	162.0	162.0	.006173	11.56	52,470	52,470	.5001	.5001	.9999	4.935
.9300	.9300	5.844	1.000	172.5	172.5	.005797	11.69	59,500	59,500	.5001	.5001	.9999	4.935
.9400	.9400	5.906	1.000	183.7	183.7	.005445	11.81	67,470	67,470	.5001	.5001	.9999	4.935
.9500	.9500	5.969	1.000	195.6	195.6	.005113	11.94	76,490	76,490	.5001	.5001	.9999	4.935
.9600	.9600	6.032	1.000	203.5	203.5	.004914	12.06	86,740	86,740	.5001	.5001	.9999	4.935
.9700	.9700	6.095	1.000	222.8	222.8	.004489	12.19	98,350	98,350	.5001	.5001	.9999	4.935
.9800	.9800	6.158	1.000	236.1	236.1	.004235	12.32	111,500	111,500	.5001	.5001	.9999	4.935
.9900	.9900	6.220	1.000	251.4	251.4	.003977	12.44	126,500	126,500	.5000	.5000	1.000	4.935
1.000	1.000	6.283	1.000	267.7	267.7	.003735	12.57	143,400	143,400	.5000	.5000	1.000	4.935

DIVISION DE EDUCACION CONTINUA
FACULTAD DE INGENIERIA U.N.A.M.

CURSO: OBRAS MARITIMAS
"HIDRAULICA MARITIMA Y DE ESTUARIOS"
DEL 11 DE MARZO AL 3 DE JUNIO
MEXICO, D.F.

O N D A S

ING. OSCAR FUENTES MARILES
JUNIO 1985

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Oscar [illegible] Mar[illegible]

HIDRAULICA MARITIMA Y DE ESTUARIOS

OBJETIVO:

Proporcionar al alumno los fundamentos teóricos de los fenómenos marítimos, con objeto de que los pueda aplicar en la solución de problemas de defensa y aprovechamiento, así como a la investigación.

PROGRAMA

1. ONDAS
   - *1.1 Ondas de pequeña amplitud*
   - *1.2 Ondas de amplitud finita*
   - *1.3 Otras teorías de ondas*

2. TRANSFORMACIONES DE LAS ONDAS
   - *2.1 Debido a profundidad somera y a amortiguamiento*
   - *2.2 Refracción*
   - *2.3 Difracción*
   - *2.4 Reflexión*
   - *2.5 Rompiente*
   - *2.6 Ascenso y descenso*

3. ONDAS OCEANICAS

   3.1 *Estadística de ondas a corto plazo*

   3.2 *Estadística de ondas a largo plazo*

   3.3 *Oleaje de diseño*

4. PREDICCION DEL OLEAJE

   4.1 *Generación y desarrollo de las olas de viento*

   4.2 *Elementos meteorológicos y factores descriptivos*

   4.3 *Métodos de predicción*

5. EFECTO DEL OLEAJE SOBRE ESTRUCTURAS MARITIMAS

   5.1 *Pilas*

   5.2 *Muros*

   5.3 *Escolleras y espigones*

6. PROCESOS COSTEROS

   6.1 *Playa*

   6.2 *Corrientes*

   6.3 *Régimen de costas*

7. ESTUARIOS

   7.1 *Hidrología de costas*

   7.2 *Cuña salina*

   7.3 *Procesos de mezclado*

   7.4 *Dispersión y sedimentación*

8. ONDAS DE LARGO PERIODO

8.1 *Oscilaciones en canales y bahías*

8.2 *Mareas*

8.3 *Otras clases de ondas*

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CAPITULO 1

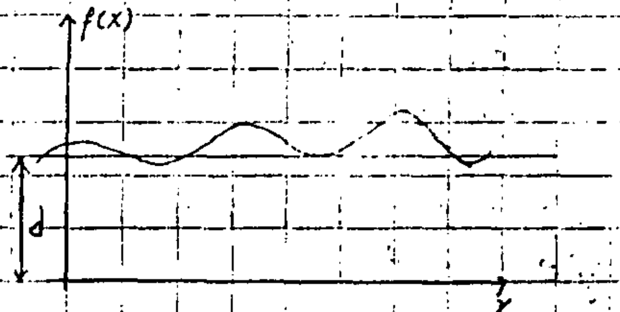
1. ONDAS

La finalidad del curso consiste en:

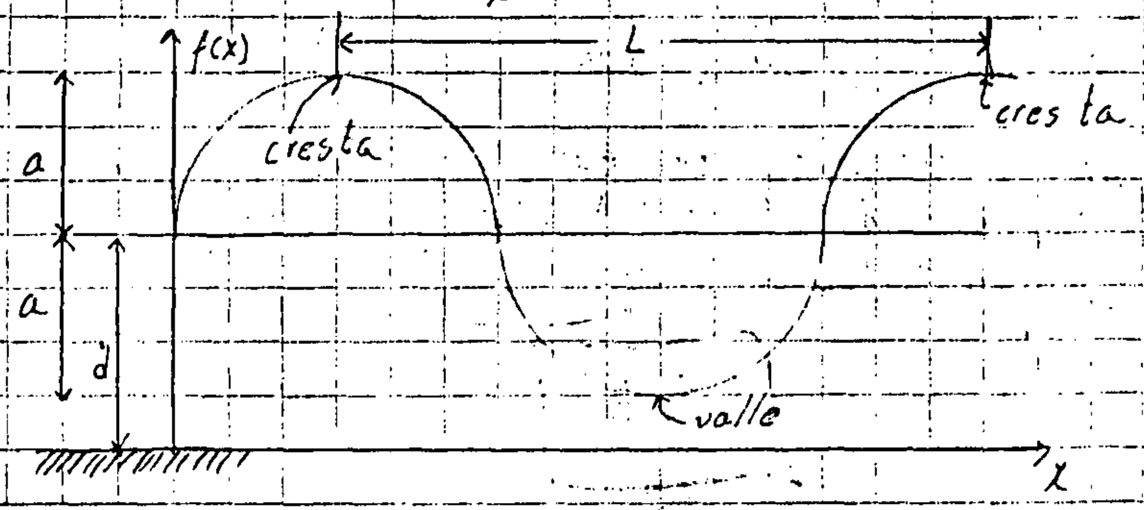
1. Entender la naturaleza de algunos fenómenos costeros, originados por oleaje y corrientes.
2. Desarrollo de técnicas de medición de las características deseadas.
3. Aclarar los mecanismos de sedimentación.
4. Investigación para lograr diseños efectivos y prácticos de construcción adecuados.
5. Entender algunos de los problemas de difusión por mezclado de agua caliente.

Introducción

El oleaje es generado por el viento, atracción del sol y la luna, ó por sismo.



En un principio, para simular el oleaje se utilizó la función trigonométrica del seno.



a, es la amplitud de la onda y es la máxima elogación que puede tener esta.

$H = 2a$, es la altura de la ola

f, es la frecuencia, se mide en (1/seg) ó hz.

El tiempo que transcurre entre el paso de dos crestas consecutivas es el periodo del oleaje

$$\boxed{T = \frac{1}{f}} \qquad 1 \leq T \leq 30 \text{ seg.}$$

(ondas de gravedad)

Para simular el oleaje se utiliza la expresión

$$f(t) = a \operatorname{sen}[2\pi f t + \phi]$$

donde ϕ es el ángulo de fase.

al oleaje simulado con esa expresión se le llama senoidal, monocromático ó armónico.

Para un instante $t = t_1$, si se toma en cuenta la longitud L del oleaje, se tiene

$$g(x) = a \operatorname{sen}(2\pi \tfrac{1}{L} t_1 + \phi)$$

para otro instante $t = t_2$, la ola ha cambiado de posición ($t_2 > t_1$), con una celeridad c que se calcula como

$$\boxed{c = \frac{L}{T}}$$

Se define como frecuencia angular a

$$\boxed{\sigma = 2\pi f = \frac{2\pi}{T}}$$

se define como número de onda (de oleaje) a

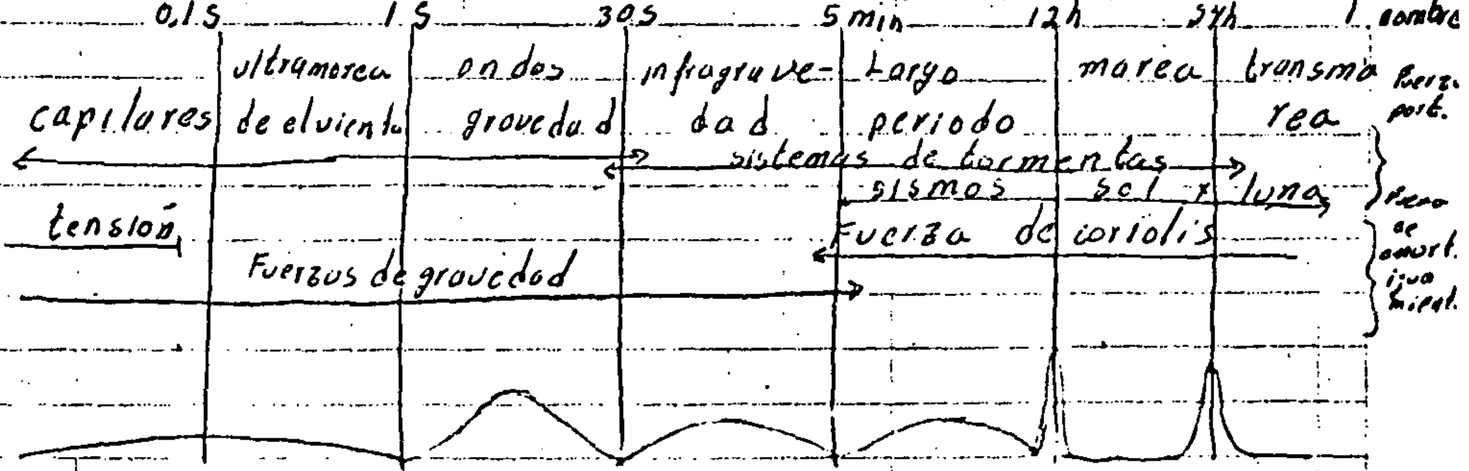
$$\boxed{k = \frac{2\pi}{L}}$$

con lo anterior se tiene que la celeridad también se puede calcular como

$$\boxed{c = \frac{\sigma}{k}}$$

Clasificación de ondas

Munk (1951), según su periodo



Clasificación del oleaje (continuación)

Características tomadas en cuenta en la clasificación	clasificación
• Altura, longitud y profundidad	ondas de pequeña amplitud (onda senoidal) ondas de amplitud finita
• Desplazamiento de la cresta	ondas estacionarias (no avanza en la dist) ondas progresivas (avanza en la dist y en c/t.)

• profundidad relativa (d/L)

	teoría	práctica
ondas en aguas profundas	$d/L \geq 1$	$d/L \geq 1/2$
ondas en aguas intermedias	$1/20 < d/L < 1$	$1/25 \leq d/L < 1/2$
ondas en aguas bajas	$d/L \leq 1/200$	$d/L \leq 1/25$

• Desplazamiento de las partículas de agua
- oscilatorio
- translatorio

• Aplicación de la fuerza perturbadora
- Libre
- forzada

• Según la teoría de oleaje
- trocoidal
- cnoidal
- solitaria

1.1 Ondas de pequeña amplitud.

(teoría lineal, oscilatoria, T. Airy).

estas ondas tienen la siguiente característica

$$a << L \quad ; \quad a << d$$

1.1.1 Teorema de Bernoulli

se parte de las ecs de Navier - Stokes.

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u + X = \frac{du}{dt} \qquad (1.a)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \nabla^2 v + Y = \frac{dv}{dt} \qquad (1.b)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \nabla^2 w + Z = \frac{dw}{dt} \qquad (1.c)$$

(ρ: densidad, p: presión, ν: viscosidad)

en el sistema anterior se tienen tres ecuaciones y cuatro incognitas

Para completar el sistema se utiliza la ecuación de continuidad.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

obtención del término $\frac{du}{dt}$

La definición de diferencial total es

$$d(\;) = \frac{\partial}{\partial x}(\;)dx + \frac{\partial}{\partial y}(\;)dy + \frac{\partial}{\partial z}(\;)dz + \frac{\partial}{\partial t}(\;)dt$$

al dividir entre dt

$$\frac{d}{dt}(\;) = \frac{\partial}{\partial x}(\;)u + \frac{\partial}{\partial y}(\;)v + \frac{\partial}{\partial z}(\;)w + \frac{\partial}{\partial t}(\;)$$

despejando a $\frac{du}{dt}$

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \qquad (3)$$

por otra parte

$$|\bar{V}|^2 = u^2 + v^2 + w^2 = V^2 \qquad (4)$$

por lo que

$$\frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}$$

de donde

$$u\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) - v\frac{\partial v}{\partial x} - w\frac{\partial w}{\partial x} \qquad (5)$$

sust. (5) en (3)

$$\frac{du}{dt} = \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) - v\frac{\partial v}{\partial x} - w\frac{\partial w}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$\frac{du}{dt} = \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) + v\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + w\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \frac{\partial u}{\partial t} \qquad (6)$$

por otra parte

$$\text{rot}\,\hat{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \hat{i}\left(\frac{\partial W}{\partial y} - \frac{\partial v}{\partial z}\right) - \hat{j}\left(\frac{\partial}{\partial x}w - \frac{\partial}{\partial z}u\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial}{\partial y}u\right)$$

$$[\text{rot}\,\hat{V}] \times \hat{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) & \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) & \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ u & v & w \end{vmatrix}$$

La componente del rotacional $\overset{de\,V}{\text{cruz}}$ V según la dirección x es

$$\underbrace{\text{rot}\,\hat{V} \times \hat{V}}_{x} = W\left(-\frac{\partial W}{\partial x} + \frac{\partial u}{\partial z}\right) - v\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (7)$$

Al comparar (6) y (7)

$$\frac{du}{dt} = \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) + \underbrace{\text{rot}\,\hat{V} \times \hat{V}}_{x} + \frac{\partial u}{\partial t} \qquad (8)$$

sust. (8) en (1.a)

$$\boxed{\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 \mu + X = \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) + \underline{\text{rot}}_x}$$

Hipótesis en la expresión de Bernoulli

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u + X = \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{V^2}{2}\right) + \underbrace{\text{rot}\,\bar{V}\times\bar{V}}_{x} \quad (9)$$

1.- $\nu = 0$; ρ es constante
2.- Existe el potencial de velocidad

$$\frac{\partial \phi}{\partial x} = u \; ; \; \frac{\partial \phi}{\partial y} = v \; ; \; \frac{\partial \phi}{\partial z} = W \quad y \quad \text{rot}\,\bar{V} = \hat{0}$$

3.- Existe un potencial de fuerzas másicas $\Omega = -gz$

$$X = \frac{\partial \Omega}{\partial x} = -\frac{\partial}{\partial x}(gz)$$

4.- p debe ser negativa ya que se trata de "compresión".

$$-\frac{\partial}{\partial x}\left(\frac{p}{\rho} + gz + \frac{V^2}{2}\right) = \frac{\partial u}{\partial t}$$

$$\frac{\partial}{\partial x}\left[\frac{p}{\gamma} + z + \frac{V^2}{2g}\right] = -\frac{1}{g}\frac{\partial u}{\partial t} \qquad (10.a)$$

de manera semejante

$$\frac{\partial}{\partial y}\left[\frac{p}{\gamma} + z + \frac{V^2}{2g}\right] = -\frac{1}{g}\frac{\partial v}{\partial t} \qquad (10.b)$$

$$\frac{\partial}{\partial z}\left[\frac{p}{\gamma} + z + \frac{V^2}{2g}\right] = -\frac{1}{g}\frac{\partial W}{\partial t} \qquad (10.c)$$

Multiplicando por dx, dy y dz, y considerando

$$\hat{ds} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$$

$$d\left(\frac{p}{\gamma} + z + \frac{V^2}{2g}\right) = -\frac{1}{g}\frac{\partial}{\partial t}\,\hat{V}\cdot\hat{ds} \qquad (11)$$

Integrando entre los puntos 1 y 2

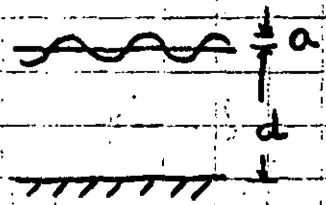
$$\left[\frac{p}{\gamma} + z + \frac{V^2}{2g}\right]_1^2 = -\frac{1}{g}\int_1^2 \frac{\partial}{\partial t}\,\hat{V}\cdot\hat{ds}$$

$$\boxed{\left[\frac{p}{\gamma} + z + \frac{V^2}{2g}\right]_1^2 = -\frac{1}{g}\int_1^2 \frac{\partial}{\partial t}\,\hat{V}\cdot\hat{ds} - \int_1^2 g\,ds} \qquad (12)$$

La expresión (12) es la forma completa de la Teorema de Bernoulli

Ondas de pequeña amplitud

$a << L \qquad a << d$



a) Bidimensional (X, Z)

b) $\frac{V^2}{2g} = 0$

Según la ec. (11)

$$d\left(\frac{p}{\gamma} + z\right) = -\frac{1}{g}\frac{\partial}{\partial t}\left(u\,dx + w\,dz\right)$$

$$d\left(\frac{p}{\gamma}+z\right)=-\frac{1}{g}\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial x}\,dx+\frac{\partial \phi}{\partial z}\,dz\right)$$

$$d\left(\frac{p}{\gamma}+z\right)=-\frac{1}{g}\frac{\partial}{\partial t}\left(d\phi\right)$$

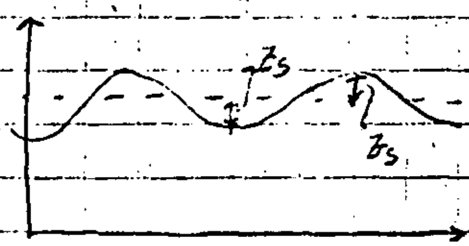
Integrando

$$\frac{p}{\gamma}+z=-\frac{1}{g}\frac{\partial \phi}{\partial t}+c'$$

$$\frac{p}{\gamma}+z+\frac{1}{g}\frac{\partial \phi}{\partial t}=c' \qquad (13)$$

En la superficie libre p es constante y que se anula con c'

$$z_s+\frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)_{z=z_s}=0 \qquad (14)$$



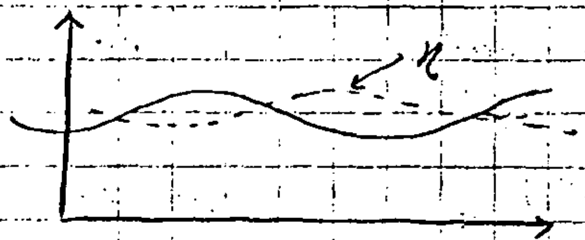
$\eta = z_s$

a).-

$$\boxed{\eta+\frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)_{z=\eta}=0} \qquad (15)$$

b). condición de frontera
La componente vertical de la velocidad (W) en la superficie es igual al cambio en el tiempo de esta superficie

$$\frac{d}{dt}\eta(x,t) = W \quad en \quad z=\eta \qquad (16)$$



$$\frac{d}{dt}\eta(x,t) = \frac{\partial}{\partial x}\eta\frac{dx}{dt} + \frac{\partial \eta}{\partial t} = W \quad en \quad z=\eta$$

$$\frac{d}{dt}\eta = \frac{\partial \eta}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \quad en \quad z=\eta \qquad (17)$$

$$* \ \frac{\partial \eta}{\partial x}\frac{\partial \phi}{\partial x} = 0$$

* serie de Taylor

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2!}\Delta x^2 + \ldots + \frac{f^{(n)}(x)\Delta x^n}{n!} + \ldots$$

así

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=\eta} = \left(\frac{\partial \phi}{\partial z}\right)_{z=0} + \cancelto{0}{\left(\frac{\partial^2 \phi}{\partial z^2}\right)_{z=0}\Delta\phi} + \cancelto{0}{\frac{\partial^3 \phi}{\partial z^3}\frac{\Delta\phi^3}{2}} + \ldots$$

o sea

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=\eta} \doteq \left(\frac{\partial \phi}{\partial z}\right)_{z=0} \qquad (18)$$

del mismo modo

y $$\left(\frac{\partial \phi}{\partial x}\right)_{z=\eta} \doteq \left(\frac{\partial \phi}{\partial x}\right)_{z=0}$$

b) $\boxed{\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial z}\right)_{t=0}}$ (19)

c) Derivando 15 parcialmente respecto al tiempo e igualando con 16

$$\frac{\partial \eta}{\partial t} + \frac{1}{g}\frac{\partial^2 \phi}{\partial t^2} = 0 \qquad \text{en } z = \eta$$

$$\boxed{\frac{\partial \phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \phi}{\partial t^2}} \qquad (20)$$

d) Como la velocidad es cíclica con respecto a la posición horizontal y el tiempo.

$$\boxed{\phi = f(z)\,\mathrm{sen}(kx - \sigma t)}$$

e) Suponiendo $\boxed{\eta = a\cos(kx - \sigma t)}$

f) Ecuación de continuidad

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad \boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0} \qquad (23)$$

g) condición de frontera en el fondo

$$\boxed{W_{z=-d} = \left(\frac{\partial \phi}{\partial z}\right)_{z=-d} = 0} \qquad (24)$$

★ Determinación de $f(z)$:

sust (21) en (23)

$$\frac{\partial \phi}{\partial x} = f(z)\, k \cos(kx - \sigma t)$$

$$\frac{\partial^2 \phi}{\partial x^2} = -f(z)\, k^2 \operatorname{sen}(kx - \sigma t)$$

$$\frac{\partial \phi}{\partial z} = f'(z) \operatorname{sen}(kx - \sigma t)$$

$$\frac{\partial^2 \phi}{\partial z^2} = f''(z) \operatorname{sen}(kx - \sigma t)$$

$$-f(z)k^2 \operatorname{sen}(kx - \sigma t) + f''(z) \operatorname{sen}(kx - \sigma t) = 0$$

$$f''(z) - k^2 f(z) = 0 \quad ; \quad f(z) = Ae^{kz} + Be^{-kz} \quad (26)$$

$$\phi = (Ae^{kz} + Be^{-kz}) \operatorname{sen}(kx - \sigma t) \quad (27)$$

procedimiento para valuar A y B (27) en (24)

$$\frac{\partial \phi}{\partial z} = (Ake^{kz} - Bke^{-kz}) \operatorname{sen}(kx - \sigma t)$$

$$(Ake^{-kd} - Bke^{kd}) \operatorname{sen}(kx - \sigma t) = 0$$

$$\boxed{Ae^{-kd} - Be^{kd} = 0} \quad (28)$$

si $\quad Ae^{-kd} = Be^{kd} = D/2$

$$A = \frac{D}{2} e^{kd} \qquad B = \frac{D}{2} e^{-kd}$$

sust (27)

$$\phi = \frac{D}{2}\left[e^{k(d+z)} + e^{-k(d+z)}\right] sen(kx - \sigma t)$$

$$\phi = D \cosh k(z+d) \, sen(kx - \sigma t)$$

Según 19

$$\eta = -\frac{1}{g}\left(\frac{\partial\phi}{\partial t}\right)_{z=0}$$

$$\frac{\partial\phi}{\partial t} = \sigma D \cosh k(z+d) \cos(kx - \sigma t)$$

$$\left(\frac{\partial\phi}{\partial t}\right)_{z=0} = -\sigma D \cosh kd \cos(kx - \sigma t)$$

$$\eta = \frac{1}{g} \sigma D \cosh kd \cos(kx - \sigma t)$$

al comparar con 22

$$\eta = a \cos(kx - \sigma t)$$

$$a = \frac{1}{g}\,\sigma D \cosh kd \quad ; \quad D = \frac{ag}{\sigma}\,\frac{1}{\cosh kd}$$

Así

$$\boxed{\phi = \frac{ag \cosh k(z+d)}{\cosh kd}\,\mathrm{sen}\,(kx - \sigma t)} \qquad (32)$$

Otros resultados de importancia

Sea la ec.

$$\phi = (Ae^{kz} + Be^{-kz})\,\mathrm{sen}\,(kx - \sigma t) \qquad (27)$$

$$\frac{\partial \phi}{\partial t} = (Ae^{kz} + Be^{-kz})\cos(kx - \sigma t)(-\sigma)$$

$$\frac{\partial^2 \phi}{\partial t^2} = -\sigma^2 (Ae^{kz} + Be^{-kz})\,\mathrm{sen}\,(kx - \sigma t)$$

en $z = 0$

$$\left(\frac{\partial^2 \phi}{\partial t^2}\right)_{z=0} = -\sigma^2 (A + B)\,\mathrm{sen}\,(kx - \sigma t)$$

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = k(A - B)\,\mathrm{sen}\,(kx - \sigma t)$$

Según 20

$$\frac{\partial \phi}{\partial z} = -\frac{1}{g}\,\frac{\partial^2 \phi}{\partial t^2}\ ;$$

$$k(A-B)\,\text{sen}(kx-\sigma t)=+\frac{\sigma}{g}\,\sigma^2(A+B)\,\text{sen}(kx-\sigma t)$$

$$\sigma^2(A+B)=+kg\,(A-B)$$

$$(\sigma^2-kg)A+(\sigma^2+kg)B=0$$

La 28 es

$$Ae^{-kd}-Be^{kd}=0$$

Lo anterior es un sistema de ec. lineal y homogéneo.

$$\begin{vmatrix}\sigma^2-kg & \sigma^2+kg\\ e^{-kd} & -e^{kd}\end{vmatrix}=+(\sigma^2-kg)e^{kd}+(\sigma^2+kg)e^{-kd}=0$$

$$\sigma^2e^{kd}-kge^{kd}+\sigma^2e^{-kd}+kge^{-kd}=0$$

$$\sigma^2(e^{kd}+e^{-kd})-kg(e^{kd}-e^{-kd})=0$$

$$\sigma^2(2\cosh kd)-kg(2\,\text{senh}\,kd)=0$$

$$\sigma^2=kg\,\frac{\text{senh}\,kd}{\cosh kd}$$

$$\boxed{\sigma^2=kg\tanh kd}$$

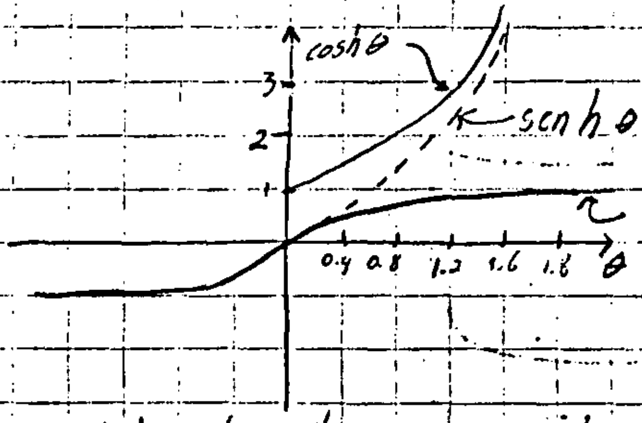
recordando que

$$\sigma = \frac{2\pi}{T} \quad y \quad k = \frac{2\pi}{L}$$

$$\frac{(2\pi)^2}{T^2} = \frac{2\pi}{L}\, g \tan h \frac{2\pi}{L} d \quad ; \quad \frac{L}{T} = \frac{T}{2\pi}\, g \operatorname{tangh} \frac{2\pi}{L} d$$

$$\boxed{C = \frac{g}{2\pi} T \operatorname{tangh} \frac{2\pi}{L} d} \qquad C = \sqrt{\frac{g}{2\pi} L \operatorname{tangh} kd}$$

Gráfica de las funciones hiperbólicas



sí $\frac{d}{L} \to \infty$

$\Rightarrow \operatorname{tangh} \frac{2\pi d}{L} \to 1$

El subíndice o indica condición de aguas profundas

$$\therefore \quad \boxed{C_0 = \frac{gT}{2\pi}} = \quad C_0 = \sqrt{\frac{g}{2\pi} L}$$

$$\boxed{L_0 = \frac{g}{2\pi} T^2 \doteq 1.56\, T^2}$$

(aproximaciones)

si $\theta \doteq 0$ $\quad \tanh \theta \doteq \theta \quad \frac{d}{L} \to 0$

si $\frac{d}{L} \leq \frac{1}{25}$ $\quad \tanh \frac{2\pi d}{L} \doteq \frac{2\pi d}{L}$

$$C = \frac{gT}{2\pi} \frac{2\pi d}{L} \qquad C = \frac{gTd}{L}$$

$$C \frac{L}{T} = gd \qquad C^2 = gd$$

$$\boxed{C = \sqrt{gd}} \text{ aguas bajas} \qquad \boxed{L = T\sqrt{gd}} \text{ aguas bajas}$$

también se deduce

$$\boxed{L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L}}$$

La expresión anterior se puede resolver por aproximaciones sucesivas o utilizando las tablas de Wiegel.

*** características del oleaje**

recordando que ϕ

$$\phi = \frac{ag}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} \operatorname{sen} h (kx - \sigma t)$$

- La <u>velocidad horizontal</u> es

$$u = \frac{\partial \phi}{\partial x} = \frac{agk}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t)$$

- La <u>velocidad vertical</u> es

$$W = \frac{\partial \phi}{\partial z} = \frac{agk}{\sigma} \frac{\operatorname{senh} k(z+d)}{\cosh kd} \operatorname{sen}(kx - \sigma t)$$

de otra manera también

$$u = \frac{agk}{\sigma} \frac{\operatorname{senh} kd}{\operatorname{senh} kd} \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t)$$

y como $c = \frac{g}{\sigma} \tanh kd$; $\tanh kd = \frac{c\sigma}{g}$

$$u = akc \frac{\cosh k(z+d)}{\operatorname{senh} kd} \cos(kx - \sigma t)$$

- otra forma alternativa es (el desplazamiento)

$$u = \frac{dx}{dt} \;;\; dx = u\,dt \;;\; x = \int u\,dt + X_0$$

<u>desplazamiento</u>

$$X = \frac{agk}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} \int \cos(kx - \sigma t)\,dt + X_0$$

$$\boxed{X = -\frac{agk}{\sigma^2} \frac{\cosh k(z+d)}{\cosh kd} \operatorname{sen}(kx - \sigma t) + X_0}$$

el desplazamiento vertical es Z

$$W = \frac{dZ}{dt} \qquad Z = \int W\,dt + Z_0$$

$$\boxed{Z = \frac{agk}{\sigma^2}\frac{\operatorname{senh} k(Z+d)}{\cosh kd}\cos(kX-\sigma t) + Z_0}$$

$$\operatorname{sen}(kX-\sigma t) = \frac{X - X_0}{-\dfrac{agk}{\sigma^2}\dfrac{\cosh k(Z+d)}{\cosh kd}}$$

$$\cos(kX-\sigma t) = \frac{Z - Z_0}{\dfrac{agk}{\sigma^2}\dfrac{\operatorname{senh} k(Z+d)}{\cosh kd}}$$

como $\operatorname{sen}^2(kX-\sigma t) + \cos^2(kX-\sigma t) = 1$

$$\left[\frac{X - X_0}{-\dfrac{agk}{\sigma^2}\dfrac{\cosh k(Z+d)}{\cosh kd}}\right]^2 + \left[\frac{Z - Z_0}{\dfrac{agk}{\sigma^2}\dfrac{\operatorname{senh} k(Z+d)}{\cosh kd}}\right]^2 = 1$$

- esta es la ec. de una elipse pero en aguas profundas las trayectorias son circulares, y en aguas intermedias tienen forma de una elipse.

de manera semejante se encuentran

$$a_x = \frac{du}{dt} \qquad a_z = \frac{dW}{dt}$$

p (presión), E (energía), $\bar{P}$ (potencia media)

- Cálculo de la Presión P.

Si $C=0$ en la ec. 13

$$\frac{P}{\gamma} + z + \frac{1}{g}\frac{\partial \phi}{\partial t} = 0 \qquad (13')$$

como $\dfrac{\partial \phi}{\partial t} = \dfrac{\partial}{\partial t}\left\{ \dfrac{ag}{\sigma} \dfrac{\cosh k(z+d)}{\cosh kd} \operatorname{sen}(kx-\sigma t) \right\}$

$$\frac{P}{\gamma} + z - a\frac{\cosh k(z+d)}{\cosh kd}\cos(kx-\sigma t) = 0$$

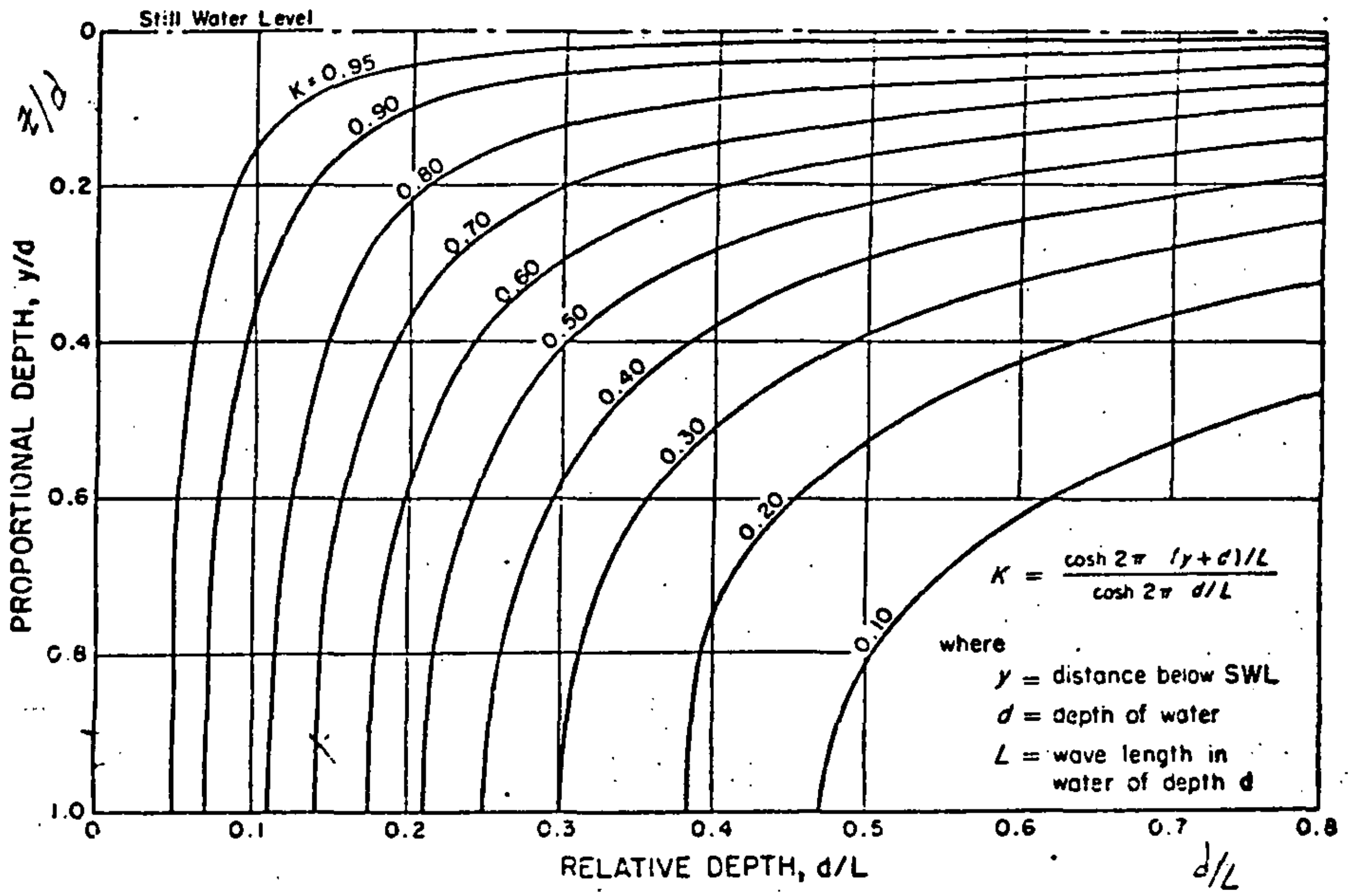
como $\eta = a\cos(kx-\sigma t)$

$$\boxed{\frac{P}{\gamma} + z = \eta\frac{\cosh k(z+d)}{\cosh kd}}$$

K es el fac de respuesta

$$K = \frac{\cosh k(z+d)}{\cosh kd}$$

Fig. 2.8. Pressure response factor K, linear theory (after Wiegel and Johnson, 1951)



★ Celeridad de un tren de ondas.

$$\eta = a\cos(kx - \sigma t)$$

$$C = \frac{L}{T} = \frac{L/2\pi}{T/2\pi} = \frac{\sigma}{k} \qquad \sigma = Ck$$

$$\eta = a\cos[k(x - ct)]$$

si $\eta = \eta_1 + \eta_2$

$$\eta = a\cos k_1(x + C_1 t) + a\cos k_2(x + C_2 t)$$

por otra parte

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\eta = 2a\cos\left[\frac{(k_1x + C_1t) + (k_2x + C_2t)}{2}\right]\cos\left[\frac{(k_1x + C_1t) - (k_2x + C_2t)}{2}\right]$$

$$\eta = 2a\cos\left[\frac{k_1 + k_2}{2}x - \frac{k_1C_1 + k_2C_2}{2}t\right]\cos\left[\frac{k_1 - k_2}{2}x - \frac{(k_1C_1 - k_2C_2)}{2}\right]$$

análisis del primer argumento.

número de onda $\frac{k_1 + k_2}{2}$

frecuencia angular $\frac{k_1C_1 + k_2C_2}{2}$

Longitud de onda $\frac{2\pi}{\frac{k_1 + k_2}{2}} = \frac{4\pi}{k_1 + k_2}$

$$\text{periodo} = \frac{4\pi}{k_1C_1 + k_2C_2}$$

análisis del segundo argumento

$$\text{número de onda} = \frac{k_1 - k_2}{2}$$

$$\text{frecuencia angular} = \frac{k_1C_1 - k_2C_2}{2}$$

$$\text{Longitud de onda} = \frac{4\pi}{k_1 - k_2}$$

$$\text{periodo} = \frac{4\pi}{k_1C_1 - k_2C_2}$$

Cuando $k_1 \doteq k_2 = k$ y $C_1 \doteq C_2 = C$

1° Arg. $L_1 = \frac{2\pi}{k}$ $T_1 = \frac{2\pi}{kC}$

representa las características de una sóla onda

2° Arg. L y T tienden a Infinito

$$C_g = \frac{\frac{4\pi}{k_1 - k_2}}{\frac{4\pi}{k_1C_1 - k_2C_2}} = \frac{k_1C_1 - k_2C_2}{k_1 - k_2} = \frac{\Delta kC}{\Delta k}$$

Sí $\Delta k \to 0$. $C_g = \frac{d(kC)}{dk}$

$$C_g = k\frac{dC}{dk} + C\frac{dk}{dk} = k\frac{dC}{dk} + C$$

recordando que

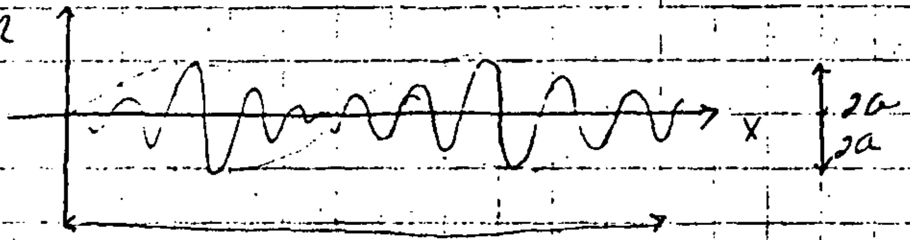
$$C = \sqrt{\frac{g}{k} \tanh kd}$$

con lo anterior se obtiene

$$C_g = \frac{1}{2}\left[1 + \frac{4\pi d}{L} \frac{1}{\operatorname{senh} 4\pi d/L}\right] C$$

ó $\boxed{C_g = n\,C}$

C_g corresponde a la velocidad de propagación de la envolvente de un tren de ondas.



$$\frac{4\pi}{k_1 - k_2} = L$$

En aguas profundas $n_0 = 0.5$
En aguas bajas $n = 1$

a n también se le llama relación de celeridades.

Tarea.: Hacer un resumen sobre teoría del oleaje de amplitud finita. Máximo dos hojas tamaño carta

Tabla 2.1 Compendio de ecuaciones de la teoría lineal

Característica de onda	Número de ec. del apéndice A	Expresión general (aguas intermedias)	Aguas profundas ($d/L > 1/2$)	Aguas poco profundas ($d/L < 1/25$)
Forma de la superficie (η)	A.3.22	$\eta = a\cos(kx-\sigma t) = a\cos\theta$		
Celeridad ($C = L/T$)	A.3.19	$C = \sqrt{\frac{g}{k}\tanh kd}$	$C = \sqrt{\frac{gL}{2\pi}}$	$C = \sqrt{gd}$
Longitud (L)	A.3.21	$L = \frac{gT^2}{2\pi}\tanh kd$	$L = \frac{gT^2}{2\pi} = 1.52T^2$	$L = T\sqrt{gd}$
Velocidad de grupo (C_g)	A.3.47	$C_g = nC = \frac{C}{2}\left[1 + \frac{2kd}{\operatorname{senh} 2kd}\right]$	$C_g = \frac{C}{2}$	$C_g = \sqrt{gd}$
Velocidad horizontal de la partícula (u)	A.3.11	$u = \frac{kga}{\sigma}\frac{\cosh[k(z+d)]}{\cosh kd}\cos\theta$	$u = \frac{a}{\sigma}e^{kz}\cos\theta$	$u = a\sqrt{\frac{g}{d}}\cos\theta$
Velocidad vertical de la partícula (w)	A.3.13	$w = \frac{kga}{\sigma}\frac{\operatorname{senh}[k(z+d)]}{\cosh kd}\operatorname{sen}\theta$	$w = \frac{a}{\sigma}e^{kz}\operatorname{sen}\theta$	$w = \frac{2a}{T}\pi\left(1+\frac{z}{d}\right)\operatorname{sen}\theta$
Aceleración horizontal de la partícula (a_x)	A.3.15a	$a_x = kga\frac{\cosh[k(z+d)]}{\cosh kd}\operatorname{sen}\theta$	$a_x = a\sigma^2 e^{kz}\operatorname{sen}\theta$	$a_x = a\sigma\sqrt{\frac{g}{d}}\operatorname{sen}\theta$
Aceleración vertical de la partícula (a_z)	A.3.15b	$a_z = -kga\frac{\operatorname{senh}[k(z+d)]}{\cosh kd}\cos\theta$	$a_z = -a\sigma^2 e^{kz}\cos\theta$	$a_z = -a\sigma^2\left(1+\frac{z}{d}\right)\cos\theta$
Desplazamiento horizontal de la partícula (ξ)	A.3.28	$\xi = -a\frac{\cosh[k(z+d)]}{\operatorname{senh} kd}\operatorname{sen}\theta$	$\xi = -ae^{kz}\operatorname{sen}\theta$	$\xi = -\frac{a}{\sigma}\sqrt{\frac{g}{d}}\operatorname{sen}\theta$
Desplazamiento vertical de la partícula (ζ)	A.3.29	$\zeta = a\frac{\operatorname{senh}[k(z+d)]}{\operatorname{senh} kd}\cos\theta$	$\zeta = ae^{kz}\cos\theta$	$\zeta = a\left(1+\frac{z}{d}\right)\cos\theta$
Presión abajo de la superficie (p)	A.3.33	$p = \gamma\eta\frac{\cosh[k(z+d)]}{\cosh kd} - \gamma z$	$p = \gamma\eta e^{kz} - \gamma z$	$p = \gamma(\eta - z)$
Energía total por unidad de área (E_T)	A.3.40	$E = \frac{\gamma a^2}{2}L$	$E = \frac{\gamma g a^2 T^2}{2\pi}$	$E = \frac{\gamma a^2}{2}T\sqrt{gd}$
Potencia por unidad de cresta promedio ($\bar{P}$)	A.3.48	$\bar{P} = \frac{nE}{T}$	$\bar{P} = \frac{\gamma g T a^2}{8\pi}$	$\bar{P} = \frac{\gamma a^2}{2}\sqrt{gd}$
Potencial de velocidad (ϕ)	A.3.10	$\phi = \frac{ga}{\sigma}\frac{\cosh k(z+d)}{\cosh kd}\operatorname{sen}\theta$	$\phi = \frac{ga}{\sigma}e^{kz}\operatorname{sen}\theta$	$\phi = \frac{ga}{\sigma}\operatorname{sen}\theta$

Compendio de ecuaciones de la teoría lineal.

★ Ondas Capilares

Cuando L se reduce y es muy pequeña empieza a tener importancia la tensión superficial

$$\left(\frac{\partial \phi}{\partial t}\right)_{z=0} + g\eta + \frac{K}{\rho}\frac{\partial^2 \eta}{\partial x^2} = 0$$

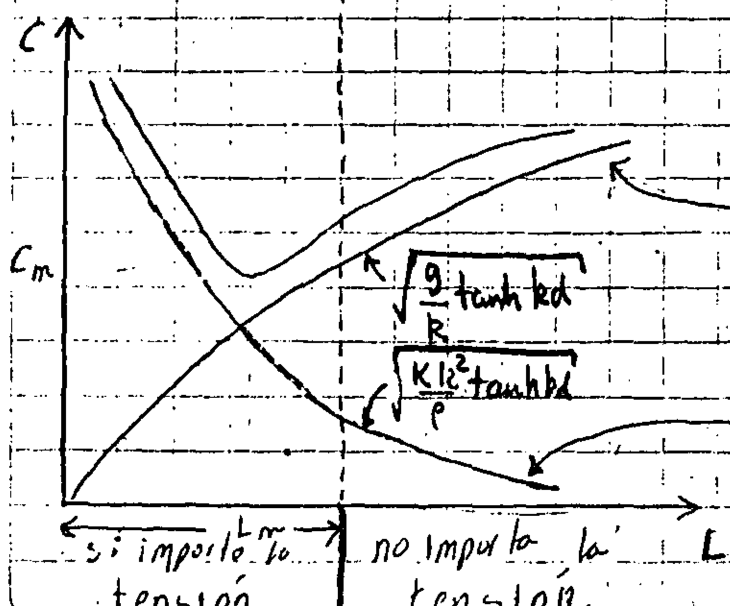
K es función la temperatura

$$\eta = a\cos(kx - \sigma t)$$

$$\frac{\partial \eta}{\partial x} = -ka\,\mathrm{sen}(kx-\sigma t)\ ;\ \frac{\partial^2 \eta}{\partial x^2} = -k^2 a\cos(kx-\sigma t) = -k^2\eta$$

$$\left(\frac{\partial \phi}{\partial t}\right)_{t=0} + \left(g - \frac{K}{\rho}k^2\right)\eta = 0$$

$$C = \sqrt{\left(\frac{g}{k} + \frac{K}{\rho}k^2\right)\tanh kd}$$



(L grande)

$$C = \sqrt{\frac{g}{k}} = \sqrt{\frac{gL}{2\pi}}$$

(L grande)

$$C = \sqrt{\frac{K}{\rho}k^2} = \sqrt{\frac{K}{\rho}}\,\frac{2\pi}{L}$$

de la figura anterior

$K = 73.5$ dinas/cm a 15°C

$C_m = 23.2$ cm/seg $\qquad L_m = 1.72$ cm

$\therefore$ No existen en la naturaleza ondas con una celeridad menor a 23.5 cm/seg

1.2 ONDAS DE AMPLITUD FINITA

recordando que

$$\eta = -\frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)_{z=\eta} \qquad (15)$$

$$\frac{\partial \phi}{\partial z} + \frac{1}{g}\frac{\partial^2 \phi}{\partial t^2} = 0 \quad \text{en } z = \eta \qquad (20)$$

$$\nabla^2 \phi = 0 \qquad (23) \qquad V^2/2g = 0$$

Tomando en cuenta que $V^2/2g \neq 0$

$$\hat{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$d\left(\frac{P}{\gamma} + z + \frac{V^2}{2g}\right) = -\frac{1}{g}\frac{\partial}{\partial t}\hat{V}\cdot \hat{ds} \qquad (11)$$

$$\bar{V} = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k} \quad ; \quad \hat{V} = \hat{\nabla}\phi$$

$$V^2 = |\bar{V}| = \hat{V}\cdot\hat{V} = \hat{\nabla}\phi\cdot\hat{\nabla}\phi$$

$$\hat{V}\cdot\widehat{ds} = \hat{\nabla}\phi\cdot\widehat{ds}$$

$$\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\hat{V}\cdot\widehat{ds} = d\phi$$

$$d\left(\frac{p}{\gamma} + z + \frac{1}{2g}\bar{\nabla}\phi\cdot\bar{\nabla}\phi\right) = -\frac{1}{g}\frac{\partial}{\partial t}d\phi$$

$$= d\left(-\frac{1}{g}\frac{\partial\phi}{\partial t}\right)$$

$$d\left(\frac{p}{\gamma} + z + \frac{1}{2g}\hat{\nabla}\phi\cdot\hat{\nabla}\phi + \frac{1}{g}\frac{\partial\phi}{\partial t}\right) = 0 \qquad (34)$$

Lo anterior es el <u>teorema de Bernoulli para la 2° Aprox</u> (caso en que $V^2/2g \neq 0$)

$$\frac{p}{\gamma} + z + \frac{1}{2g}\hat{\nabla}\phi\hat{\nabla}\phi + \frac{1}{g}\frac{\partial\phi}{\partial t} = C_1 \qquad (34')$$

$$p = 0 \quad \text{en} \quad z = \eta$$

$$\eta = -\frac{1}{g}\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}\bar{\nabla}\phi\cdot\bar{\nabla}\phi\right)_{z=\eta} + C_1 \qquad (36)$$

se usa en lugar de B15

C. De varias identidades matematicas

$$\frac{dP}{dt} = \frac{\partial P}{\partial x}\underbrace{\left(\frac{dx}{dt}\right)}_{u} + \frac{\partial P}{\partial y}\underbrace{\left(\frac{dy}{dt}\right)}_{v} + \frac{\partial P}{\partial z}\underbrace{\left(\frac{dz}{dt}\right)}_{w} + \frac{\partial P}{\partial t}\underbrace{\frac{dt}{dt}}_{1}$$

$$\frac{1}{\rho}\frac{dP}{dt} = \frac{1}{\rho}\left[\frac{\partial P}{\partial x}u + \frac{\partial P}{\partial y}v + \frac{\partial P}{\partial z}w + \frac{\partial P}{\partial t}\right]$$

$$\frac{d}{dt}\left(\frac{P}{\rho}\right) = \frac{\partial}{\partial x}(P/\rho)u + \frac{\partial}{\partial y}(P/\rho)v + \frac{\partial}{\partial z}(P/\rho)w + \frac{\partial}{\partial t}(P/\rho)$$

$$= \frac{\partial \phi}{\partial x}\frac{\partial (P/\rho)}{\partial x} + \frac{\partial \phi}{\partial y}\frac{\partial}{\partial y}(P/\rho) + \frac{\partial \phi}{\partial z}\frac{\partial (P/\rho)}{\partial z} + \frac{\partial}{\partial t}(P/\rho)$$

$$= \hat{\nabla}\phi \cdot \hat{\nabla}(P/\rho) + \frac{\partial}{\partial t}\left(\frac{P}{\rho}\right)$$

$$\frac{d}{dt}\left(\frac{P}{\rho}\right) = \left[\hat{\nabla}\phi \cdot \hat{\nabla} + \frac{\partial}{\partial t}\right]\left(\frac{P}{\rho}\right) \qquad (36)$$

multiplicando 34 por g y ordenando

$$\frac{P}{\rho} = -\left(zg + \frac{1}{2}\hat{\nabla}\phi \cdot \hat{\nabla}\phi + \frac{\partial \phi}{\partial t}\right) + C_2 \qquad (37)$$

sustituyendo 37 en 36

$$\left[\hat{\nabla}\phi \cdot \hat{\nabla} + \frac{\partial}{\partial t}\right]\left[-zg - \frac{1}{2}\hat{\nabla}\phi\,\hat{\nabla}\phi - \frac{\partial \phi}{\partial t} + C_2\right] = \frac{d}{dt}\left(\frac{P}{\rho}\right)$$

$$g\left[\hat{\nabla}\phi\cdot\hat{\nabla}+\frac{\partial}{\partial t}\right]z+\frac{1}{2}\left[\hat{\nabla}\phi\cdot\hat{\nabla}+\frac{\partial}{\partial t}\right](\hat{\nabla}\phi\cdot\hat{\nabla}\phi)+$$

$$+\left[\hat{\nabla}\phi\cdot\hat{\nabla}+\frac{\partial}{\partial t}\right]\frac{\partial\phi}{\partial t}=-\frac{d}{dt}\left(\frac{p}{\rho}\right)$$

$$g\left[\frac{\partial\phi}{\partial x}\frac{\partial}{\partial x}+\frac{\partial\phi}{\partial z}\frac{\partial}{\partial z}+\frac{\partial}{\partial t}\right]z+\frac{1}{2}\left[\nabla\phi\cdot\nabla+\frac{\partial}{\partial t}\right](\hat{\nabla}\phi\cdot\hat{\nabla}\phi)+$$

$$\hat{\nabla}\phi\cdot\hat{\nabla}\left(\frac{\partial\phi}{\partial t}\right)+\frac{\partial^2\phi}{\partial t^2}=-\frac{d}{dt}\left(\frac{p}{\rho}\right)$$

$$g\frac{\partial\phi}{\partial z}+g\frac{\partial z}{\partial t}+\frac{\partial^2\phi}{\partial t^2}+\frac{1}{2}\left[\vec{\nabla}\phi\cdot\nabla+\frac{\partial}{\partial t}\right](\hat{\nabla}\phi\cdot\hat{\nabla}\phi+\hat{\nabla}\phi\cdot\hat{\nabla}\left(\frac{\partial\phi}{\partial t}\right)=$$

$$=\frac{d}{dt}\left(\frac{p}{\rho}\right)$$

en $z=0$, $p=0$, $\frac{\partial p}{\partial t}=0$, $\frac{\partial z}{\partial t}=0$

$$\underbrace{\left(g\frac{\partial\phi}{\partial z}+\frac{\partial^2\phi}{\partial t^2}\right)}_{1^\circ\ \text{Orden}}+\underbrace{\hat{\nabla}\phi\cdot\hat{\nabla}\left(\frac{\partial\phi}{\partial t}\right)}_{2^\circ\ \text{Orden}}+\underbrace{\frac{1}{2}\left[\hat{\nabla}\phi\cdot\hat{\nabla}+\frac{\partial}{\partial t}\right](\hat{\nabla}\phi\cdot\hat{\nabla}\phi)}_{3^\circ\ \text{Orden}}=0$$

Terminos de: 1° Orden 2° Orden 3° Orden

1.2.1. TEORIA DE STOKES

Sea el potencial de velocidades

$$\phi(x,z)=-Cx-C\beta e^{kz}\operatorname{sen}kx$$

y la función línea de corriente

$$\psi(x,z)=-Cz+C\beta e^{kz}\cos kx$$

} para aguas profundas

Proponiendo

$$\eta = \beta e^{k\eta} \cos kx$$

como

$$e^{\theta} = 1 + \theta + \frac{1}{2!}\theta^2 + \frac{1}{3!}\theta^3 + \frac{1}{4!}\theta^4 + \ldots$$

así

$$\eta = \beta\left[1 + k\eta + \frac{1}{2}k^2\eta^2 + \frac{1}{6}k^3\eta^3 + \ldots\right]\cos kx$$

sust. (34) y (42) en (35)

$$\left(\frac{2g}{C^2} - 2k\right)\eta + k^2\beta^2 e^{2k\eta} = C_3 \qquad (45)$$

desarrollando en series $e^{2k\eta}$, sust (43), y despreciando terminos de orden 3 en adelante

$$\left(\frac{2g}{C^2} - 2k - 2k^3\beta^2\right)\eta + (2k^4\beta^2)\eta^2 = C_4$$

$$\frac{2g}{C^2} - 2k + 2k^3\beta^2 = 0 \qquad 2k^4\beta^2 = 0$$

proponiendo

$$\eta = \beta\eta_0 + \beta^2\eta_1 + \beta^3\eta_2$$

$$\eta = \beta\eta_0 + \beta^2\eta_1 + \beta^3\eta_2 \qquad (48)$$

sust. en 44

$$\eta = \beta[1 + k\eta + 1/2\, k^2\eta^2 + 1/6\, k^3\eta^3 + 1/24\, k^4\eta^4]\cos kx$$

$$\beta\eta_0 + \beta^2\eta_1 + \beta^3\eta_2 = \beta[1 + \beta k\eta_0 + \beta^2 k\eta_1 + \beta^2(k^2 1/2\,\eta_0)]\cos kx$$

$$= \beta[1 + \beta k\eta_0 + \beta^2 k\eta_1 + \beta^2(1/2\, k^2\eta_0)]\cos kx$$

igualando los coeficientes

$$\eta_0 = \cos kx$$

$$\eta_1 = k\eta_0\cos kx = k\cos^2 kx = \frac{1}{2}k + \frac{1}{2}k\cos 2kx$$

$$\eta_2 = (k\eta_1 + 1/2\, k\eta_0)\cos kx$$

$$\eta_2 = \frac{9}{8}k^2(\cos kx + \frac{3}{8}\cos 3kx$$

$$\eta = \beta\cos kx + \beta^2\frac{k}{2}(1 + \cos 2kx) + \frac{9}{8}(\beta^3 k^2(\cos kx + \frac{3}{8}\cos 3kx)$$

$$\eta = \frac{1}{2}k\beta^2 + \underbrace{\beta(1 + \frac{9}{8}k^2\beta^2)}_{a}\cos kx + \frac{1}{2}k\beta^2\cos 2kx + \frac{3}{8}k^2\beta^3\cos 3kx$$

se propone

$$\beta = \varepsilon_1 a + \varepsilon_2 a^2 + \varepsilon_3 a^3$$

$$\varepsilon_1 = ? \quad ; \quad \varepsilon_2 = ? \quad ; \quad \varepsilon_3 = ?$$

$$a = \beta(1 + \frac{9}{8}\beta^2 k^2)$$

$$a = (\varepsilon_1 a + \varepsilon_2 a^2 + \varepsilon_3 a^3)(1 + \frac{9}{8}[\varepsilon_1 a + \varepsilon_2 a^2 + \varepsilon_3 a^3]^2 k^2)$$

$$a = a\varepsilon_1 + a^2\varepsilon_2 + a^3(\varepsilon_3 + 9/8\, k^2\varepsilon_1^2) + \ldots$$

$$1 = \varepsilon_1 \quad ; \quad 0 = \varepsilon_2 \quad ; \quad 0 = \varepsilon_3 + \frac{9}{8}k^2\varepsilon_1^2 = \varepsilon_3 + \frac{9}{8}k^2$$

$$\beta = 1a + 0a^2 + \left(-\frac{9}{8}k^2\right)\overset{\doteq 0}{a^3} \qquad \beta = a$$

$$\boxed{\eta = a\cos kx + \frac{a^2 k}{2}(1 + \cos 2kx) + \frac{9}{8}a^3k^2(\cos kx + \frac{3}{8}\cos 3kx)}$$

esta es la ecuación de la superficie libre según la 3° aproximación de STOKES (aguas profundas)

$$\left[\ \eta = a\cos kx \qquad \text{(teoría lineal)}\ \right]$$

— · —

$$\frac{2g}{C^2} - 2k + 2k^3\beta^2 = 0 \qquad (46)$$

mult. por $C^2/2k$

$$\frac{g}{k} - C^2 + C^2k^2\beta^2 = 0 \quad ; \quad C^2 = \frac{g}{k} + C^2k^2\beta^2$$

se propone

$$C^2 = b_0 + b_1\beta + b_2\beta^2 + b_3\beta^3 \qquad b_i = ?$$

$$b_0 + b_1\beta + b_2\beta^2 + b_3\beta^3 = \frac{g}{k} + (b_0 + b_1\beta + b_2\beta^2 + b_3\beta^3)k^2\beta^2$$

$$b_0 = \frac{g}{k} \;;\; b_1 = 0 \;;\; b_2 = k^2 b_0 = k^2\frac{g}{k} = kg$$

$$b_3 = 0 \;;\quad C^2 = \frac{g}{k} + 0\beta + kg\beta^2 + 0\beta^3$$

$$\boxed{C^2 = \frac{g}{k}\left(1 + k^2\beta^2\right)} \qquad \left(C^2 = \frac{g}{k}(1 + k^2a^2)\right.$$

esta es la ecuación de la celeridad en aguas profundas según la tercera aproximación de STOKES.

Para la 1° Aprox (en aguas profundas) $C^2 = g/k$

STOKES 2° Aprox.

$$\eta = a\cos(kx - \sigma t) + \frac{\pi H^2}{8L}\frac{\cos[2\pi d/L(2 + \cosh 4\pi d/L]}{\operatorname{sen} h^3(2\pi d/L)}\cos 2(kx - \sigma t)$$

$$C = \frac{gT}{2\pi}\tanh\frac{2\pi d}{L} \;;\quad L = \frac{gT^2}{2\pi}\tanh\frac{2\pi d}{L}$$

3° Aproximación . STOKES.

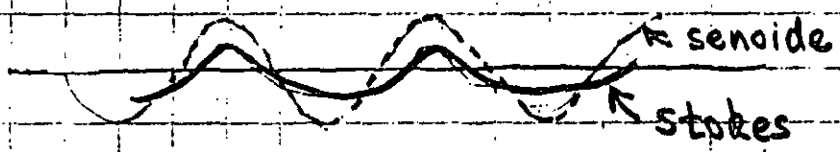
$$\eta = a\cos(kx-\sigma t) + \frac{\pi^2 a^2}{L} f_2\left(\frac{d}{L}\right)\cos 2(kx-\sigma t) + \frac{\pi^3 a^3}{L^2} f_3\left(\frac{d}{L}\right)$$
$$\cos 3(kx-\sigma t)$$

$$f_2\left(\frac{d}{L}\right) = \frac{(2+\cos 2kd)\cosh kd}{2\,\mathrm{senh}^3 kd}$$

$$f_3\left(\frac{d}{L}\right) = \frac{3}{16}\;\frac{1+8\cosh^6 kd}{\mathrm{senh}^6 kd}$$

$$c^2 = \frac{gL}{2\pi}\,\tanh kd\left[1+\left(\frac{\pi a}{L}\right)^2 \frac{8+\cosh 4kd}{8\,\mathrm{senh}^4 kd}\right]$$

Los aproximaciones son para $\frac{1}{10} \leq \frac{d}{L} \leq \frac{1}{8}$



2° Aproximación $\quad \theta = kx - \sigma t$

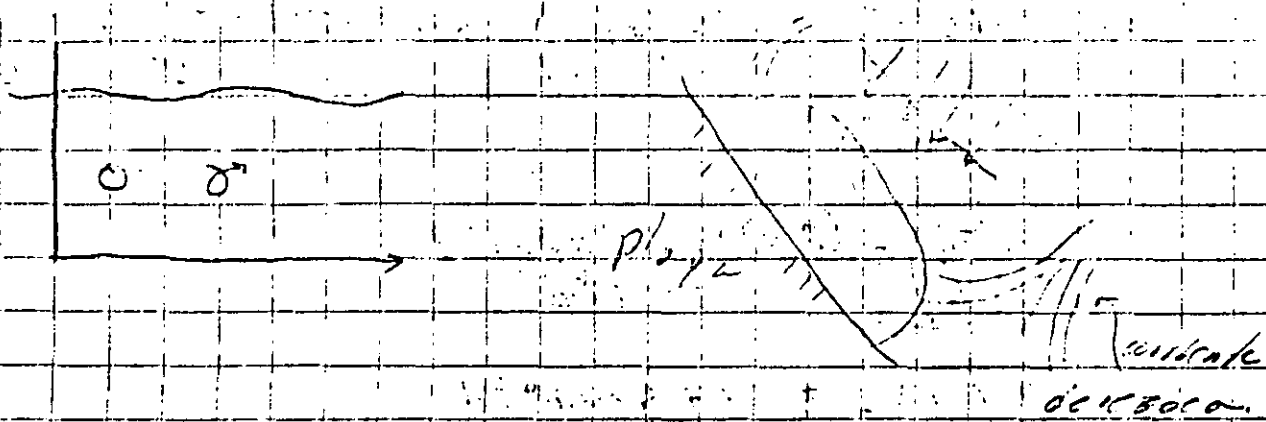
$$\xi = -\frac{HgT^2}{4\pi L}\,\frac{\cosh k(z+d)}{\cos kd}\,\mathrm{sen}\,\theta + \frac{\pi H^2}{8L}\,\frac{1}{\mathrm{senh}^2 kd}\left\{1 \mp \frac{3}{2}\,\frac{\cos 2k(z+d)}{\mathrm{sen}^2 kd}\right\}$$

$$+\left(\frac{\pi H}{L}\right)^2 \frac{c}{2}\, t\; \frac{\cosh 2k(z+d)}{\mathrm{senh}^2 kd}$$

¡ES UN TERMINO NO PERIODICO!

La distancia desplazada durante un tiempo igual al periodo T dividido entre T se conoce como transporte de masa

Corrientes de retorno o de rezaca



★ Transporte de masa

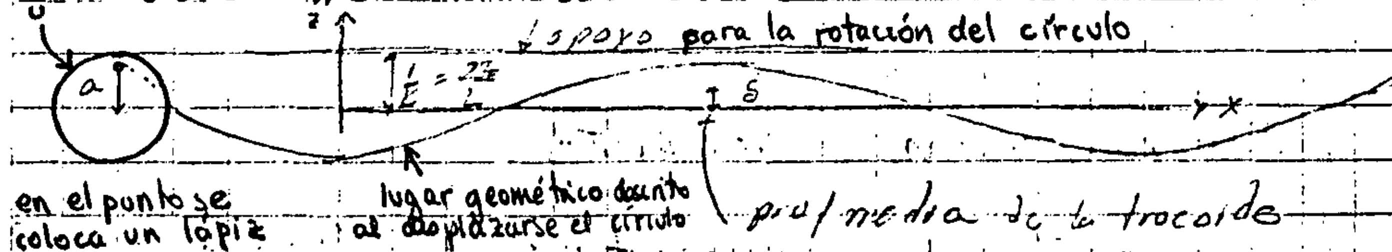
Desplazamiento

$$U(z) = \frac{\left(\frac{\pi H}{L}\right) \frac{C\cancel{T}}{2} \frac{\cosh 2k(z+d)}{\operatorname{senh}^2 kd}}{\cancel{T}}$$

1.3 Otras teorías de ondas

• Teoría TROCOIDAL. (aguas profundas)

se utiliza la ecuación de la trocoide

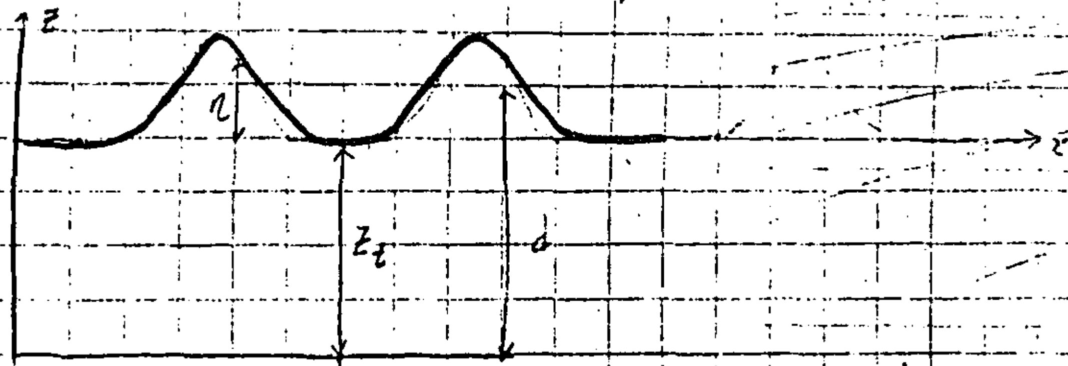


$$\delta = \frac{\pi H^2}{4L}$$

δ es la que hay entre el eje de las absisas y la profundidad media

• Teoría de las ondas Cnoidales

sirve para aguas poco profundas e intermedias.

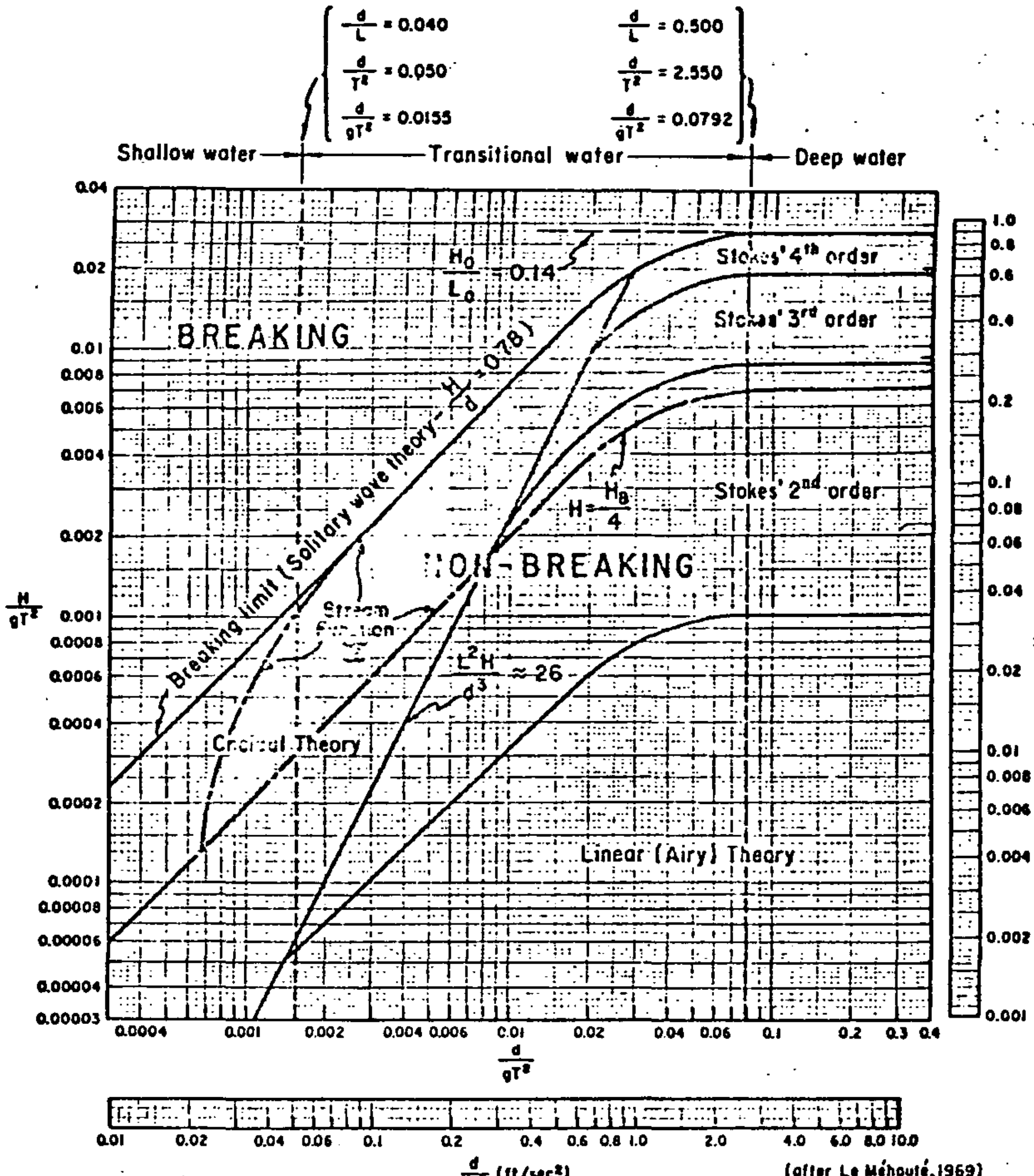


$$\eta = z_t + H cn^2 [2K(k)(x/L - t/T), k]$$

$cn(\ )$: función coseno cnoidal

$$K(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k\,sen^2 x}} \quad \text{integral elíptica}$$

Regiones de aplicación de las teorías de oleaje



EJEMPLOS

Ejemplo 1.

Se desea determinar si las pilas de un muelle se verán afectadas por la erosión cuando se presentan olas de 4 m de altura y 8 seg. La profundidad del fondo es 16 m y el material del mismo resiste hasta velocidades de 2.5 m/seg.

La solución consiste en encontrar la velocidad horizontal u en el fondo.

u es máximo cuando $\cos(kx - \sigma t) = 1$

Datos: $H = 4$ m, $T = 8$ seg, $d = 16$ m, $V = 2.5$ m/seg,

solución.

$$L_o = \frac{gT^2}{2\pi} = \frac{9.81(8)^2}{2\pi} \doteq 100 \text{ m} \quad \text{(para aguas profundas)}$$

$$\text{con } \frac{d}{L_o} = \frac{16}{100} = 0.16 \Rightarrow \frac{d}{L} = 0.1917$$

$$L = \frac{16}{0.1917} = 83.46 \text{ m}$$

$$u = \frac{kga}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t)$$

$$\text{para } \begin{matrix} z = -d \\ x = t = 0 \end{matrix} \quad u = \frac{kga}{\sigma} \frac{1}{\cosh kd}$$

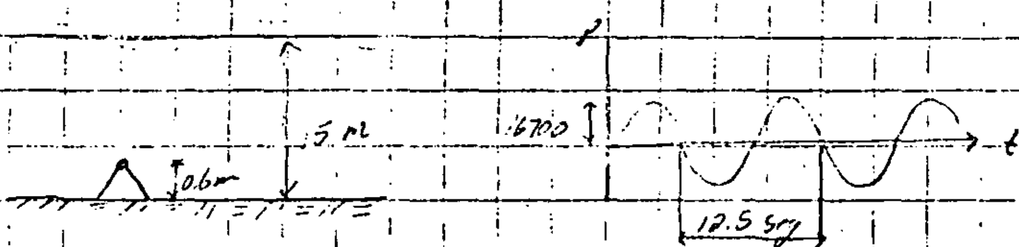
$$k = \frac{2\pi}{L} = \frac{2\pi}{83.46} = 0.0753 \; ; \; \sigma = \frac{2\pi}{T} = \frac{2\pi}{8} = 0.7854$$

$$\cosh kd = \cosh(0.0753)(16) = 1.817$$

$$u_{max} = \frac{0.0753(9.81)\,2}{0.7854} \frac{1}{1.817} = 1.035 \text{ m/seg}$$

como $u_{max} < u_{res}$ (no hay erosión)

2.- Un traductor de presión localizado a 0.6 m del fondo en un lugar donde la profundidad del agua es 15 m. El aparato ha registrado una presión máxima de 16 700 kg/m² con una frecuencia de 0.08 seg⁻¹. Encuentre la altura de la ola.



$$P = \gamma a \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t) - \gamma z$$

$$L_0 = 1.561\,(12.5)^2 = 243.95 \text{ m}$$

$$\frac{d}{L_0} = \frac{15}{243.95} = 0.0615 \qquad \frac{d}{L} = 0.1058 \; ; \quad L = 141.78 \text{ m}$$

$$k = \frac{2\pi}{L} = \frac{2\pi}{141.78} = 0.0443$$

$$16700 = 1013 \frac{H}{2} \frac{\cosh (0.0443)(-14.4+15)}{\cosh (0.0443)(15)} - 1013(-14.4)$$

$$H = 16700 - (1013)(14.4) \frac{2}{1013} \frac{(1.2248)}{1.0003} = 5.63 \text{ m}$$

El factor de respuesta de la presión es

$$K = \frac{\cosh(k)(z+d)}{\cosh kd} = \frac{1.0003}{1.2248} = 0.817$$

3.- A una profundidad de 30 m se tiene un oleaje de periodo de 12 seg. y altura 5 m. Encuentre a) la longitud de la ola, b) la celeridad, c) la energía total por unidad de longitud de cresta

Datos: $d = 30\,m$, $T = 12\,seg$, $H = 5\,m$

Solución:

a) Como no se sabe si se esta en la condición de aguas profundas ó no, se utiliza la siguiente expresión y se resuelve por aproximaciones sucesivas (es convergente y recursiva)

$$L = \frac{g T^2}{2\pi} \tanh \frac{2\pi d}{L}$$

se propone $L = 177.04\ m$

$$L = \frac{9.81\,(12)^2}{2\pi} \tanh \frac{2\pi(30)}{177.04} = 177.04\,m$$

$$\therefore \boxed{L = 177.04\,m}$$

como $\frac{d}{L} = \frac{30}{177.04} = 0.17$ y $0.5 > 0.17 > 0.04$

se esta en aguas intermedias.

b) $$C = \sqrt{\frac{g}{k} \tanh kd} = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}}$$

$$C = \sqrt{\frac{9.81(177.04)}{2\pi} \tanh \frac{2\pi(30)}{177.04}} \qquad \therefore \boxed{C = 14.75\,\frac{m}{seg}}$$

otra manera sencilla de resolver el segundo inciso es

$$c = \frac{L}{T} = \frac{177.04}{12} = 14.75 \text{ m/seg}$$

c) con $\gamma = 1.025 \text{ ton/m}^3$ y $a = H/2 = 5/2 = 2.5 \text{ m}$

$$Et = \frac{\gamma a^2 L}{2} = \frac{(1.025)(2.5)^2(177.04)}{2}$$

$$\therefore \boxed{E_T = 567.09 \frac{\text{ton}\cdot\text{m}}{\text{m}}}$$

4. Sea una onda de 300m de largo y 10m de altura, donde la profundidad del fondo es 150 m. Calcule: a) la celeridad de la onda, b) los desplazamientos de las partículas de agua en el fondo y c) Compare la celeridad con la de aguas poco profundas.

Datos: $L = 300 \text{ m}$; $H = 10 \text{ m}$, $d = 150 \text{ m}$

Solución:

$$\frac{d}{L} = \frac{150}{300} = 0.5$$ se esta en aguas profundas (Lim.)

a) $$C_0 = \sqrt{\frac{gL}{2\pi}} = \sqrt{\frac{9.81(300)}{2\pi}} = 21.64 \text{ m/seg}$$

$$\therefore \boxed{C_0 = 21.64 \text{ m/seg}}$$

b) $$\xi = -a e^{kz} \operatorname{sen}\theta = -\frac{H}{2} e^{\frac{2\pi d}{L}} \operatorname{sen}(kx - \sigma t)$$

(para desplazamiento horizontal)

como $C_o = \frac{L}{T_o}$; $T_o = \frac{L}{C_o} = \frac{300}{21.64}$ $\boxed{T_o = 13.86 \text{ seg}}$

para desplazamiento vertical

$\zeta = a e^{kz} \cos\theta = \frac{H}{2} e^{\frac{-2\pi d}{L}} \cos(kx - \sigma t)$

$k = \frac{2\pi}{L} = \frac{2\pi}{300} = 0.0209$; $\sigma = \frac{2\pi}{T} = \frac{2\pi}{13.86} = 0.4533$

para $\underline{x = 0}$

t (seg)	ξ (m)	ζ (m)
T/4	0.2161	0.0
T/2	0.0	-0.2161 ✗ 0
3/4 T	-0.2161	0.0
T	0.0	0.2161 ✗ 0

✓

c) para aguas poco profundas

$\frac{d}{L} < \frac{1}{25}$ $d < \frac{300}{25} = 12$ m

$C = \sqrt{g d} = \sqrt{9.81(12)}$ $C = 10.85$ m/seg

$\therefore \boxed{C_o > C}$ ✓

5. ¿ Cuales son: a) las velocidades orbitales y b) los desplazamientos máximos a una profundidad de 10m, de un oleaje de 8 seg., 4 m de altura y donde la profundidad del fondo está a 20m

Datos: $T = 8$ seg, $H = 4$ m, $d = 20$ m, $z = -10$ m

Solución:

Calculo de la longitud L

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L}$$

$$L = \frac{9.81(8)^2}{2\pi} \tanh \frac{2\pi(20)}{L}$$

$$\boxed{L = 88.79 \text{ m}}$$ ✓

$$\frac{d}{L} = \frac{20}{88.79} = 0.23$$

como $0.5 > 0.23 > 0.04$

se esta en aguas intermedias. ✓

a) Cálculo de la velocidad horizontal para $z = -10$ m

$$u = \frac{kga}{\sigma} \frac{\cosh[k(z+d)]}{\cosh kd} \cos(kx - \sigma t)$$

$$k = \frac{2\pi}{L} = \frac{2\pi}{88.79} = 0.07 \;;\; \sigma = \frac{2\pi}{T} = \frac{2\pi}{8} = 0.79 \;;\; a = \frac{H}{2} = 2 \text{ m}$$

La velocidad vertical de la particula es

$$W = \frac{kga}{\sigma} \frac{\operatorname{senh}[k(z+d)]}{\cosh kd} \operatorname{sen}(kx - \sigma t)$$

para $\begin{cases} z = -10 \text{ m} \\ x = 0 \text{ m} \end{cases}$

t (seg)	u (m/seg)	W (m/seg)
T/4	0.0	-0.6229
T/2	-1.0316	0.0
3T/4	0.0	0.6229
T	1.0316	0.0

✓

b) Cálculo de los desplazamientos máximos

- desplazamiento horizontal

$$\xi = -a \frac{\cosh[k(z+d)]}{\operatorname{senh} kd} \operatorname{sen}(kx - \sigma t)$$

es máximo cuando $\operatorname{sen}(kx - \sigma t) = \pm 1$

$$\xi_{máx} = -2 \frac{\cosh[0.07(-10+20)]}{\operatorname{senh}(0.07)(20)} (\pm 1) = \underline{\pm 1.30 \text{ m}}$$

- desplazamiento vertical

$$\zeta = a \frac{\operatorname{senh}[k(z+d)]}{\operatorname{senh} kd} \cos(kx - \sigma t)$$

es máximo cuando $\cos(kx - \sigma t) = \pm 1$

$$\zeta_{máx} = 2 \frac{\operatorname{senh}[0.07(-10+20)]}{\operatorname{senh}(0.07)(20)} (\pm 1) = \underline{\pm 0.7930 \text{ m}}$$

6. En un canal de oleaje monocromático se ha obtenido por medio de mediciones la información contenida en la fig 1. En la línea punteada marcada con t=0 s. se señala el punto donde empezó a funcionar el cronómetro. Si la profundidad del agua es de 1.6m. Determine: a) la celeridad, periodo y frecuencia del oleaje. b) Las aceleraciones de las partículas de agua. c) La expresión matemática para la superficie de la ola. d) la potencia promedio por unidad de longitud de cresta. e) La presión en el punto B ubicado a $X = 1m$ y $Z = -0.4m$ (fig 1), para $t = 0$, $t = 0.46$ seg, $t = 0.92$ seg. f) compare los resultados de e) con respecto con las presiones cuando no hay oleaje, desde luego para el mismo punto B.

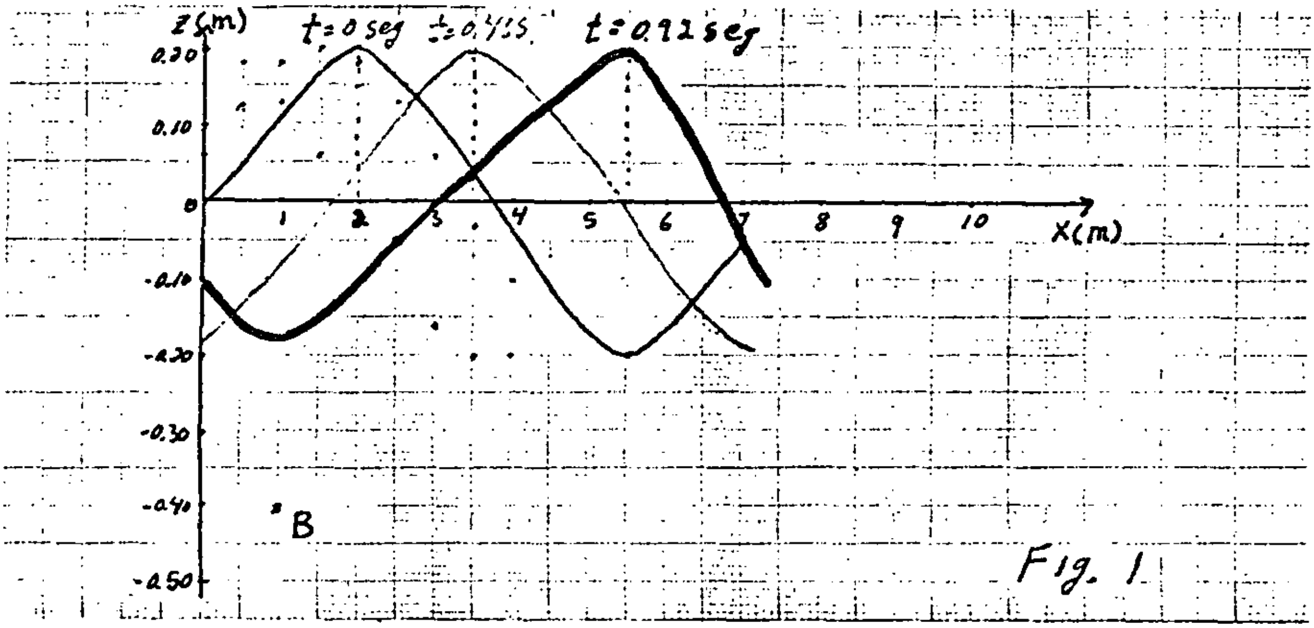


Fig. 1

Datos: $d = 1.6$ m

a) de la figura (1)

$a = 0.20\ m$; $H = 2a = 0.40\ m$

$L \doteq 7.5 m$; $\dfrac{0.46\ seg}{1.5\ m} = \dfrac{T}{7.5}$ $T = 2.3\ seg.$

solución

a) La celeridad se puede calcular con la siguiente fórmula

$$C = \sqrt{\frac{g}{k} \tanh kd} \quad ; \quad k = \frac{2\pi}{L}$$

$$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} = \sqrt{\frac{9.81(7.5)}{2\pi} \tanh \frac{2\pi(1.6)}{7.5}}$$

$$\therefore \boxed{C = 3.195 \text{ m/seg}}$$

como $\frac{C}{} = \frac{L}{T}$; $T = \frac{L}{C} = \frac{7.5}{3.195} = \quad \therefore \boxed{T = 2.347 \text{ seg}}$

y la frecuencia f es $f = \frac{1}{T} = \frac{1}{2.347}$ $\boxed{f = 0.426 \text{ seg}^{-1}}$

Revisión se debe cumplir que

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L}$$

$$7.5 = \frac{9.81(2.347)^2}{2\pi} \tanh \frac{2\pi(1.6)}{7.5} \quad ; \quad 7.5 \doteq 7.499 \quad \text{(si cumple).}$$

También se observa que el periodo obtenido de la figura y el perido obtenido con las expresiones es aproximadamente igual

$T_o = 2.3$ (observado)

$T_e = 2.347$ seg (estimado)

b) Las aceleraciones de las partículas de agua en $z = -d/2 = -0.80\,m$ son

.- dirección horizontal

$$a_x = kga \frac{\cosh[k(z+d)]}{\cosh kd} \operatorname{sen}(kx - \sigma t)$$

$d = 1.6\,m$; $k = \frac{2\pi}{L} = 0.8377$; $z = -0.8\,m$ $T = 2.347\,seg$

.- dirección vertical

$$a_z = -kga \frac{\operatorname{sen} h[k(z+d)]}{\cosh kd} \cos(kx - \sigma t)$$

para $x = 0$

t	a_x	a_z
$T/4$	-0.9930	0.0
$T/2$	0.0	0.5810
$3/4\,T$	0.9930	0.0
T	0.0	-0.5810

c) La expresión es

$$\eta = 0.20 \cos 2\pi(x/1.5 - t/2.347)$$

para $t = 0$

x	η
0	0.20
0.5	0.18
1	0.13
1.5	0.06
2	-0.02
2.5	-0.10
3	-0.16
3.5	-0.20
4	-0.20

para $t = 0.46\,seg$

x	η
0	0.06
0.5	0.13
1	0.18
1.5	0.20
2	0.18
2.5	0.13
3	0.06
3.5	-0.03
4	-0.10

ver fig 1

d) La potencia promedio por unidad de longitud de cresta

$$\bar{P} = \frac{nE}{T} = \frac{\gamma a^2}{2}\frac{L}{T}\,\frac{1}{2}\left(1 + \frac{2kd}{\operatorname{senh} 2kd}\right)$$

$$\bar{P} = \frac{(1.025)(0.20)^2}{2}\frac{(7.5)}{2.347}\,\frac{1}{2}\left(1 + \frac{2(2\pi/7.5)1.6}{\operatorname{senh} 2(2\pi/7.5)1.6}\right)$$

$$\therefore \boxed{\bar{P} = 0.0 \quad \frac{ton\,m}{m\,seg}}$$

e) La presion en el punto B segun t es

$$p(t) = \gamma\eta\frac{\cosh[k(z+d)]}{\cosh kd} - \gamma z$$

$$p(0) = 1.025(0.2)\cos((2\pi/7.5)(1) - (2\pi/2.347)0)\frac{\cosh \frac{2\pi}{7.5}(-0.4+1.6)}{\cosh \frac{2\pi}{7.5}(1.6)} - 1.025(-0.40) =$$

$$\boxed{p(0) = 0.5141 \ ton/m^2}$$

$$p(0.46) = 1.025(0.2)\cos((2\pi/7.5)(1) - (2\pi/2.347)(0.46))\frac{\cosh 2\pi/7.5(-0.4+1.6)}{\cosh 2\pi/7.5(1.6)} - 1.025(-0.40) =$$

$$\boxed{p(0.46) = 0.5531 \ ton/m^2}$$

$$p(0.92) = 1.025(0.2)\cos((2\pi/7.5)(1) - (2\pi/2.347)(0.92))\frac{\cosh 2\pi/7.5(-0.4+1.6)}{\cosh \frac{2\pi}{7.5}(1.6)} - 1.025(-0.40) =$$

$$\boxed{p(0.92) = 0.4015 \ ton/m^2}$$

f) La presion en B cuando no hay oleaje es

$p = \gamma z = 1.025(0.40) = 0.41$ ton/m^2

$\underline{p < p(0)}$ $\underline{p \doteq p(0.92)}$ ✓

$\underline{p < p(0.46)}$

7. Demostrar que la energía total por unidad de longitud de cresta es

$$E = E_c + E_p = \frac{\gamma H^2 L}{16} + \frac{\gamma H^2 L}{16} = \frac{\gamma H^2 L}{8} \qquad (1)$$

.- Cálculo de E_c

$$E_c = \frac{\gamma}{2g}\int_0^L\int_{-d}^0 (u^2 + w^2)\, dx\, dz \qquad (2)$$

donde $u^2 = (kga)^2 \frac{\cosh^2 k(z+d)}{\cosh^2 kd} \operatorname{sen}^2(kx - \sigma t)$ ... (a)

$$w^2 = (-kga)^2 \frac{\cosh^2 k(z+d)}{\cosh^2 kd} \cos^2(kx - \sigma t) \qquad (b)$$

sustituyendo a y b en 2 queda

$$E_c = \frac{\gamma}{2g}\int_0^L\int_{-d}^0 \left[\left(\frac{kga}{\sigma}\right)^2 \frac{\cosh^2 k(z+d)}{\cosh^2 kd} \operatorname{sen}^2(kx - \sigma t) + (-kga)^2 \frac{\cosh^2 k(z+d)}{\cosh^2 kd} \times \cos^2(kx - \sigma t)\right] dx\, dz$$

$$E_c = \frac{\gamma}{2g}\int_0^L\int_{-d}^0\left(\frac{gka}{\sigma}\right)^2\frac{1}{\cosh^2 kd}\left[\cosh^2 k(z+d)\cos^2(kx-\sigma t)+\right.$$

$$\left.+\operatorname{senh}^2 k(z+d)\operatorname{sen}^2(kx-\sigma t)\right]dx\,dz$$

$$E_c = \frac{\gamma}{2g}\left(\frac{gka}{\sigma}\right)^2\frac{1}{\cosh^2 kd}\,\frac{\pi}{k^2}\,(\operatorname{sen} kd \cos kd)$$

y como $\sigma^2 = kg \tanh kd$

$$\therefore\ E_c = \frac{\gamma a^2}{4}L$$

- Cálculo de E_p

$$E_p = \gamma\int_0^L\int_0^\eta z\,dx\,dz = \gamma\int_0^L \left.\frac{z^2}{2}\right|_0^\eta dx = \gamma\int_0^L \frac{\eta^2}{2}dx$$

Como $\eta = a\cos(kx-\sigma t)$

$$E_p = \frac{\gamma}{2}\int_0^L a^2\cos^2(kx-\sigma t)\,dx = \frac{\gamma a^2}{2}\left(\frac{\pi}{k}\right)$$

como $k = 2\pi/L$

$$E_p = \frac{\gamma a^2}{4}L$$

recordando que $E_T = E_c + E_p$

finalmente

$$\boxed{E_T = \frac{\gamma a^2}{2}L}$$

8.- Demostrar que la potencia por unidad de longitud de cresta es

$$\bar{P} = \frac{n E}{T}$$

Se define como potencia de una onda a la energía de la onda por unidad de tiempo propagada en la dirección de recorrido de la onda.

La potencia es $P = \int_0^{-d} (p + \gamma z) u \, dz$

pero $p + \gamma z = \gamma \eta \frac{\cosh k(z+d)}{\cosh kd}$

y $\eta = a \cos(kx - \sigma t)$

$$P = \int_0^{-d} \left[\gamma a \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t) \right] \left[\frac{gak}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - \sigma t) \right] dz$$

$$P = \frac{\gamma a^2 g}{\sigma} \frac{\cos^2(kx - \sigma t)}{\cosh^2 kd} \int_0^{-d} \cosh^2 k(z+d) k \, dz$$

$$P = \frac{\gamma a^2 g}{\sigma} \frac{\cos^2(kx - \sigma t)}{\cosh^2 kd} \left[\frac{kd}{2} + \frac{\operatorname{senh} 2kd}{4} \right]$$

La potencia promedio durante T es $\bar{P}$

$$\bar{P} = \frac{1}{T} \int_0^T P \, dT$$

$$\bar{P} = \frac{1}{T}\frac{\gamma a^2 g}{\sigma^2 \cosh^2 kd}\left[\frac{kd}{2} + \frac{\operatorname{senh} 2kd}{4}\right]\int_0^T \cos^2(kx-\sigma t)\,\sigma\, dt$$

$$\bar{P} = \frac{1}{T}\frac{\gamma a^2 g}{\sigma^2 \cosh^2 kd}\left[\frac{kd}{2} + \frac{\operatorname{senh} 2kd}{4}\right]\pi$$

recordando que $\frac{g}{\sigma} = \frac{c}{\tanh kd}$

$$\bar{P} = \frac{1}{T\sigma}\frac{\gamma a^2 g}{\tanh kd}\frac{\pi}{\cosh^2 kd}\left[\frac{kd}{2} + \frac{\operatorname{senh} 2kd}{4}\right]$$

$$\bar{P} = \frac{\gamma a^2 c}{\operatorname{senh} 2kd}\left[\frac{kd}{2} + \frac{\operatorname{senh} 2kd}{4}\right]$$

$$\bar{P} = \gamma a^2 c\,\frac{1}{4}\left[1 + \frac{2kd}{\operatorname{senh} 2kd}\right]$$

$$\bar{P} = \frac{\gamma a^2}{2}\,\frac{L}{T}\,\frac{1}{2}\left[1 + \frac{2kd}{\operatorname{senh} 2kd}\right]$$

como $\frac{C_g}{C} = n = \frac{1}{2}\left[1 + \frac{2kd}{\operatorname{senh} 2kd}\right]$ y $E = \frac{\gamma a^2}{2}L$

finalmente

$$\boxed{\bar{P} = \frac{n E_T}{T}}$$ ✓